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THE

PHYSICAL REVIEW.

THE NATURE OF THE PROCESS OF IONIZATION OF GASES BY ALPHA RAYS.

By R. A. Millikan, V. H. Gottschalk and M. J. Kelly.

Synopsis.

Ionization of gases by α-rays; valency of the ions.—By catching an ionized molecule upon an oil drop at the instant of ionization and then measuring the charge thus added to the drop, it has been directly proved that the ionization of an air molecule by either β-, γ-, or X-rays uniformly consists in the detachment of one electron. In the case of ionization by α-rays, the authors show by an extended discussion of previous work bearing on the subject that while some experiments indicate the possibility of the formation of multi-valent ions, none of them is conclusive. Therefore, the droplet method was used to supply direct evidence. The necessary modifications of apparatus are described and the experimental procedure. Uncovered radium furnished the α-rays. The gases tested were *air, carbon dioxide, carbon tetrachloride, methyl iodide,* and *mercury dimethyl;* so that α-rays were shot through atoms of H, C, O, N, Cl, I and Hg, which vary in atomic weight from 1 to 200. *Experimental results.* Altogether 2,900 ions were each caught on an oil drop at the instant of ionization, and each had its charge determined. Of these captures, only 5 corresponded to a double charge and no ion was caught which carried three or more elementary charges. How many of the 5 double charges obtained were due to doubly-charged ions and how many to simultaneous catches of two singly charged ions, is unknown. In any case the results prove that, at least 99 times out of 100, ionization by an α-ray, in the case of each of the gases and vapors studied, consists in the detachment of a single electron from a molecule. The observations for 170 captures by three droplets in three gases are given in detail. The paper ends with a *discussion* of the circumstances under which multi-valent ions may possibly be produced by α-rays.

I. Introduction.

BY directly catching upon an oil drop at essentially the instant of ionization the residue of the ionized atom and then measuring the charge thus communicated to the drop, Millikan and Fletcher[1] obtained the direct proof that the act of ionization of air by β and γ rays of radium, as well as by X-rays, uniformly consists in the detachment from a neutral molecule of one single negative electron. The extension of the same

[1] R. A. Millikan and Harvey Fletcher, Phil. Mag. (6), 21, 753 (1911).

method to the determination of the nature of the act of ionization produced by α rays is more difficult and more interesting. It is more difficult because it is necessary to make the observations at a pressure so low as to render negligibly small the chance that two nearby molecules, separately ionized by a given α particle, may be thrown by the field simultaneously upon a given drop. It is more interesting because the enormously powerfully-ionizing α particle, with its relatively huge mass and strong field, might be expected to knock off in some instances more than one electron from a given atom. The number thus detached might well depend upon the complexity and chemical characteristics of the atom traversed. Indeed J. J. Thomson's results with canal rays discussed below made more than likely the discovery of multi-valent ions resulting from α ray ionization.

II. Preceding Work Bearing upon this Problem.

The only preceding work dealing directly with the ionization by α rays is that by Townsend and his pupils, and that by Franck and Westphal.

In 1908 Townsend[1] devised a method for measuring directly the ratio u/D, where u denotes the mobility of the ion and D its coefficient of diffusion. Utilizing the equation $ne = (u/D)P$, where n is the number of molecules in a cubic centimeter of gas at a pressure of P dynes per square centimeter, he could obtain the value of ne. If the ions measured were *all* univalent, the value of ne should be 1.23×10^{10} electrostatic units, while if a fraction of them had multiple charges, the value would be greater. Haselfoot[2] modified the Townsend method which was used for X-rays, so that he could determine the value of ne for the ions of air when radium was the ionizing agent. For positive ions he obtained values ranging from 1.24×10^{10} to 1.37×10^{10} electrostatic units; for the negative ions when he used dry air he could not obtain values consistent with the Townsend theory of the experiment, but when moisture was admitted, the results indicated that all ions were univalent. Haselfoot[3] repeated these experiments three years later, using the pressure and field strengths found most suitable in his previous work. Of the experiments made, the seven which gave results closest to the mean were made some with positive and some with negative ions. The mean value of ne deduced from these was 1.22×10^{10}, the value given by each experiment not differing from the mean by more than 5 per cent. These experiments then, so far as they go, give no indication of multiple charges produced *in air* by α rays.

[1] J. S. Townsend, Proc. Roy. Soc., A, 81, 464 (1908).
[2] C. E. Haselfoot, Proc. Roy. Soc., A, 82, 18 (1909).
[3] C. E. Haselfoot, Proc. Roy. Soc., A, 87, 350 (1912).

Franck and Westphal[1] determined *ne* for ions produced in air by polonium, which gives only α particle ionization. They determined the coefficients of mobility and of diffusion separately and found from their results no indications that α particles produce other than univalent ions in air. The conclusion is based on diffusion measurements which differ among themselves by 7 per cent.

The conclusion from both of these investigations is that they furnish no evidence that multi-valent ions are produced in air by α particles. It has been pointed out however[2] that such measurements have to do with the charges carried by ions at a considerable time after their formation and, because of the possibility of recombinations, do not necessarily justify a conclusion as to the valency of the ions actually formed in the ionization process.

Wilson's[3] well-known photographs of the paths of α particles throw no real light on this question, since he obtained no photographs in which all the drops in a known length of the cloud trail left by the α particle could be counted. However, by a modification of the Wilson expansion apparatus, Bumstead[4] has obtained photographs of the tracks of α particles in hydrogen, and has found distinct evidence of electronic trails radiating from the paths of the α particles. He attributes these to ionization by swift δ rays liberated from the molecules by the α particles. Thus evidence is furnished that a considerable portion of the ionization by α particles takes place in this indirect way. As we know that swiftly moving corpuscles ionize by liberating only one electron from an atom, we should expect from this evidence that at least a large portion of the ions would be univalent. Indeed, even if the evidence obtained from experiments on mobilities and diffusion coefficients were altogether precise and unambiguous, *the Bumstead experiments would leave quite open the question as to what happens to the atom when the alpha particle itself goes through it.*

Nor is any decisive evidence on this point found in Geiger's[5] measurement of the total ionization produced by α rays per mm. of path, although these experiments deserve consideration in this connection. In air at atmospheric pressure and at 12° C. Geiger found the number of ions produced by α rays of radium *C* in traversing the first millimeter to be 2250, while the number produced per mm. of path at the end of about 65 mm. was 7600. Now the mean free path of air under these condi-

[1] J. Franck and W. Westphal, Verh. d. Deutsch. Phys. Ges., 11, 276 (1909).

[2] R. A. Millikan, loc. cit.

[3] C. T. R. Wilson, Proc. Roy. Soc., A, 87, 277 (1912).

[4] H. A. Bumstead, PHYS. REV. (2), 8, 715 (1916).

[5] Geiger, Proc. Roy. Soc., 82, page 486 (1909). Also see page 164 of Rutherford's "Radioactive Substances and their Transformations" (1913).

tions is .0001 mm. Hence introducing the factor 4 to take account of the fact that the dimensions of the α ray are negligible in comparison with the diameter of the atom, and the factor $\sqrt{2}$ to allow for the fact that the velocity of agitation of the molecules of air is negligible in comparison with the speed of the α ray, it will be seen that the number of molecules penetrated by an α particle in going 1 mm. through air under the aforesaid conditions is

$$\frac{1}{.0001 \times 4\sqrt{2}} = 1{,}750.$$

Hence it is clear from Geiger's results that the α ray produces at all points in its path a number of ions (unit charges) which is larger than the number of molecules through which it passes—1.3 times as large at the beginning, and 4.5 times as large near the end of the path. Whether the whole of this excess is due to the delta rays brought to light by Bumstead's experiments, or whether the alpha rays sometimes form multivalent ions is beyond the power of Geiger's method to determine, or indeed of any method which does not separate sharply the different types of ions, if such exist, from one another.

Sir J. J. Thomson,[1] however, from his investigations of canal rays has brought forward good reasons for concluding that positively charged atoms or molecules may ionize a molecule of gas through which they are moving, by two distinct methods. They may, first, pluck a single corpuscle out of an atom or molecule past, or through, which they are moving. This process would, of course, produce only univalent positive ions. They may, second, collide with the atom as a whole with enough energy so that "if there were several corpuscles connected with about the same firmness to the atom, the result of the atom acquiring a high velocity in a collision might be the liberation of all the corpuscles and the production of a multiply charged atom." Thomson found large numbers of such multivalent atoms for every element investigated, with the notable exception of hydrogen. The number of charges liberated appeared to be independent of any chemical properties of the atom, although an increasing function of the atomic weight: thus, nitrogen showed two; oxygen, two; neon, three; krypton, four or five; mercury, eight.

In this work of Thomson then, we find conclusive evidence for the formation of ions with more than one charge. In the case of mercury Thomson brings forward good evidence to support the view that the process of ionization always consists in the detachment from the neutral atom of mercury either of one corpuscle or else of eight corpuscles. He thinks that the parabolas which he found corresponding to 2, 3, 4, 5, 6

[1] Sir. J. J. Thomson, Phil. Mag. (6), 24, 671 (1912). Also Positive Rays, pages 37, 38, 52 and 74.

and 7 charges on the mercury atom appeared because some of the eight lost-charges had been regained before the ion entered the crossed magnetic and electric fields, the action of which on the moving ion produced these parabolas. Further, by comparing the intensity of the parabola corresponding to one charge with the sum of the intensities of the parabolas corresponding to 2, 3, 4, 5, 6 and 7 charges, we can gain some notion of the relative numbers of atoms which, from this point of view, acquire one charge and eight charges respectively in the act of ionization. Such a study reveals the fact that though the number of multivalent ions formed in Thomson's experiments was perhaps a small fraction of the number with one charge, it was not at all negligibly small. The sum of the intensities of the doubly and trebly valent argon parabolas is apparently quite comparable with the intensity of the univalent argon parabola (see Plate XV., Fig. 2, Phil. Mag., 24, 1912). Similarly in the case of mercury, parabolas 2, 3, 4, 5, 6 and 7 (Fig. 28, Rays of Positive Electricity) sum up to an intensity not of a wholly different order from parabola 1. Thomson says:[1] "The ratio of the number of atoms which have only one charge to that of those which have two or more charges is very variable and depends on conditions which are not yet fully understood. For example in the case of the carbon atom this ratio seems to depend to a very great extent on the type of gaseous carbon compound in the discharge tube. With some hydrocarbons the doubly charged carbon atoms are relatively much brighter than with others. Again, in the case of oxygen I have found that the purer the oxygen the fainter was the line due to the doubly charged oxygen atom in comparison with that due to the atom with only one charge. It would thus seem that atoms torn from chemical compounds were more likely to have a double charge than those obtained from a molecule of the element. Chemical combination can not, however, be the only means by which the atoms acquire multiple charges, for the atoms of the inert monatomic gases, neon, argon and crypton, are remarkable for the ease with which they acquire multiple charges." When it is recalled that in Thomson's experiments not only positive rays, but in addition β and X-rays are acting as powerful ionizing agents, and that these latter practically always produce singly charged positives, it will be seen that the inference is justified that a considerable part of the ionization produced by the positive rays corresponds to the production of multivalent ions.

These experiments of Thomson's then quite justified the expectation that since alpha rays are only a particular kind of positive rays, they would in many instances produce multivalent ions.

Further, the method of attack herein taken, consisting as it does of

[1] Thomson, Rays of Positive Electricity, page 53.

catching the residue of the individual atoms at practically the instant at which the act of ionization takes place, should enable decisions to be reached upon certain moot questions upon which the positive ray method of Thomson was incapable of giving an unambiguous answer. In particular,

1. Can the act of ionization of the mercury atom by a central impact from a positive ray be definitely proved to consist of the detachment of eight corpuscles as Thomson inferred? If so, our method should reveal positive residues carrying eight charges. Thomson observed no parabola corresponding to more than seven charges. He *inferred* that eight had been produced, but that 1, 2, 3, etc., had been regained in forming the ion which produced the parabolas corresponding to 7, 6, 5, etc., charges.

2. Does a positive ray which, instead of making a central impact, passes through the outer regions of an atom, ever detach two or more electrons in so doing? Such a result is to be expected, and, although the positive ray experiments present no evidence in its favor, they also do not present conclusive evidence to the contrary. How does this effect, if it exists, depend upon the atomic weight of the atom?

III. EXPERIMENTAL WORK.

The type of apparatus previously described[1] was used in the present work, with, however, certain important changes. A narrow beam of α particles passing immediately underneath the oil drop was obtained by placing about 0.1 milligram of radium bromide in a small tin box, 6 mm. wide by 2 mm. high by 12 mm. deep, which fitted tightly into a cylindrical lead plug held at the proper height and placed just at the edge of the condenser. (See Fig.) A slit, made in the ebonite sheet encircling the condenser, corresponded to the opening of the tin box. A small door of lead in a brass frame, operated by a crank from without the partially exhausted chamber, allowed the beam of α particles to enter the condenser when desired. To lessen difficulties due to drift, it was found expedient, especially with vapors, to close also the small slit in the ebonite by a spring attachment to the lead door of the radium container.

It was of course recognized from the first that it is difficult to distinguish between the practically simultaneous advent upon a drop of two or more separate ions and the advent of a multiply-charged ion having the same total charge. To eliminate this uncertainty, a sufficiently small quantity of radium was placed at such a distance from the oil drop that the probability of two α particles crossing the space beneath the drop at the same instant was very small. This probability was further diminished

[1] R. A. Millikan, PHYS. REV., 32, 349 (1909).

by using very small drops. For the drop of average size, if the α particles from 0.1 milligram of radium bromide were emitted at regularly spaced intervals of time and equally in all directions, one α particle a minute would have passed under the drop. Finally the chance that a single α particle would pass through two different molecules of the gas both of which were immediately beneath the drop was made as small as possible by working at low pressures and using small drops. These precautions could be sufficiently well taken to insure that the probability of simultaneous catches from these causes was vanishingly small. But the "induced radioctivity," discussed later, introduced another source of simultaneous catches, and this unwelcome interference was only under partial control.

The vessel was exhausted to pressures ranging from 4 to 10 centimeters of mercury, a number of oil drops were produced (by means of a "capillary tube atomizer" designed by us for use at these pressures), and a very small one of these drops, holding a positive charge, was suspended in the upper part of the field between A and B, Fig. 1, by adjusting the field

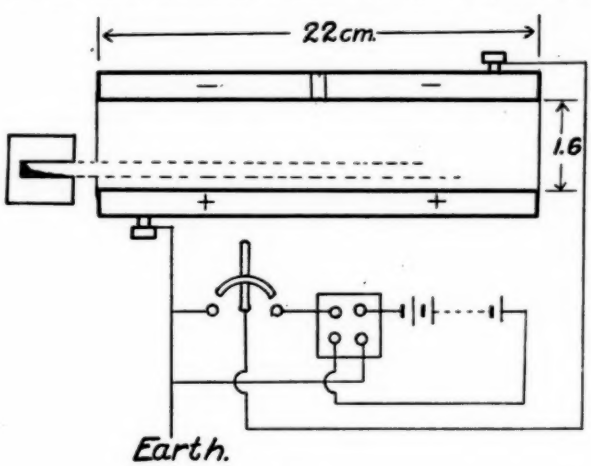

Fig. 1.

strength until the drop was nearly balanced. The lead door of the radium container was then opened so as to allow a stream of α particle to shoot underneath the drop. A molecule beneath the drop, when ionized, is thrown immediately upwards by the field, impinges on the drop and increases its positive charge. The question of the valency of the ion may at once be settled by computation from the observed change in the speed of the drop.

That only those ions which are formed beneath the drop, contribute to the changes in speed was very effectively brought into evidence several

times when improper alignment of the beam from the radium made it impossible to make captures other than those due to the "induced radioactivity," even with the radium door open. Proper alignment at once corrected this defect, so that the conclusion is fully warranted that the great majority of the catches recorded were due to the beam of α rays passing immediately beneath the drop. This part of the experiment also shows the complete untenableness of the position taken by Frank and Westphal[1] relative to Millikan and Fletcher's work on X-ray ionization, namely that a possible reason for their failure to observe doubles was the slow rate of diffusion of doubles. Neither in those experiments nor in these can diffusion play any rôle whatever, for the catches are all made in fields so powerful that an ion is thrown to the appropriate plate within something like a ten-thousandths of a second of the instant at which it is formed, and one having a double charge would move with still greater speed.

For convenience in the measurements of speed, a scale containing 80 divisions was placed in the focal plane of the observing microscope. The procedure followed was, to select a drop with the proper speed under gravity, and then to get a unit positive charge upon it by opening the radium door and throwing the field on and off until the proper charge was obtained, and finally to adjust the field for an approximately balanced condition. The radium door was then opened until a sudden start in the drop showed that an ion had been caught. The radium door was then immediately closed and the time required for the passage of the drop over ten divisions of the scale, observed. The door was again opened until another sudden increase in the speed indicated a second capture; the door again closed and the speed taken as before. When the drop came too close to the upper plate, the field was reversed (with the radium door closed) and the drop pulled down to the desired distance above the lower plate, when the field was again reversed and observations continued. Occasionally, the drop would make a catch as it was being drawn down from the upper plate. This was due either to "induced radioactivity" or to stray accidental ionization. It was decided at the beginning that when such a catch did occur, it would be considered a break in the series of observations and would not be counted; for, first, such a catch cannot come from the source under investigation; and, second, the observer needs the brief interval for rest. After the advent of the second or third charge, the speed had usually increased so much that further changes were more difficult to observe and at this point the field was always decreased to such a value that the next two or three catches produced

[1] J. Franck and W. Westphal, Phil. Mag. (6), 22, 547 (1911).

easily observed changes in speed. In some of the observations, two such changes were made. After the number of positive charges on the drop had reached five, this excess was removed by throwing off the field for a fraction of a second and opening the radium door for an instant. Proceeding in this way, in a very few trials a unit positive charge could be obtained upon the drop. It is then an easy matter to bring the drop back to unit charge, or to any other desired number of charges and then to repeat the series of operations detailed above.

The same drop was usually kept under observation for a long interval of time, in one instance for three hours, long series of catches upon it observed (160 on drop No. 10 in air), and the "valency" of the charged molecule in the case of each capture determined

As indicated in preceding papers the determination of the valency is very simple, since for a given drop, at a particular pressure and in a constant field, there is a definite speed for the drop when it carries 1, 2, 3 or n charges. Indeed the recurrence of these definite speeds scores and, in some instances, hundreds of times furnishes a most striking confirmation of the atomic structure of electricity and demonstrates the non-existence in all these experiments of subelectrons. In this work no attempt was made to obtain a precise determination of the speeds, since great accuracy was not necessary for the purposes of the investigation. A stop-watch was used throughout and timings taken only over the small distance represented by the ten divisions of the microscope scale. Neither was great precaution taken to obtain dust-free air, which has been shown in preceding work to be essential in obtaining the value of e wth accuracy. We are concerned only with the *relative* values of the charges carried by a given drop after successive captures of ions from the gas under investigation. These relative values can be determined from the equation:

$$ne = \frac{mg}{Fv_1}(v_1 + v_2),$$

where v_1 is the velocity of the droplet downward under gravity,
v_2 is its velocity upward under the field,
m is its mass,
e is the elementary charge,
F is the field intensity,
n is the number of charges on the drop.

Substituting for v_1 and v_2 respectively d/t_1 and d/t_2 where d is the distance between ten divisions of the reading microscope, and t_1 and t_2 are the seconds required for the droplet to traverse the distance d under gravity and under the field respectively, and substituting also the

expression pN for F where N is the number of trays of storage batteries employed (the storage batteries were built in trays of about 160 volts and a whole number of trays was always used) and p is the volts per tray divided by the distance between the condenser plates, we readily obtain:

$$n = \frac{mg}{pe}\left(1 + \frac{t_1}{t_2}\right)\frac{1}{N}.$$

In all of the records we have tabulated the quantity $\left(1 + \frac{t_1}{t_2}\right)\frac{1}{N}$ which is proportional to the number of charges on the droplet. If we divide this quantity by the greatest common divisor of the series of its values for any drop, we obtain the number of charges. This greatest common divisor was in every case also an observed quantity, since the unit charge on each droplet was identified before beginning the series of observations. The results of observations on the various gases and vapors are given below.

IV. AIR.

After the radium had been enclosed in the apparatus for a few days it was found that the frequency of catching was increasing to such a degree that there could be no certain differentiation between succeeding catches which were almost simultaneous and possible multi-valent ions. Also, with the door of the radium container closed, a series of observations exactly similar to those with the door open could be taken. Even when the radium was removed, the rate of capture was found to be about the same as that anticipated from the radium. Series of captures with different drops were observed in this way over a period of three weeks in which time 620 captures were recorded with seventeen drops. *Of these 620, there was only one capture which was either a doubly charged ion or two singly charged ions captured simultaneously.*

The rate of capture was undiminished at the end of one month and it was concluded that the interior of the apparatus had become coated with the active deposit of slow transformation, radium $D + E + F$, which was first studied by Rutherford[1] who found the curve of activity practically straight for the first few months. There was probably present also some "recoil radiation of radium," as the activity seemed too great to ascribe to the active deposit of slow transformation alone.

While the greater portion of the foregoing captures were undoubtedly due to α particles, the rate of capture was entirely uncontrollable and it was thought best therefore to return to the original plan of producing the ionization by a single beam of α rays. Polonium was unfortunately

[1] E. Rutherford, Phil. Trans., A, 204, 169 (1905).

Table I.

Drop No. 2 in Air.

Pressure, 9.75 cms. Time of run, 2:00 hours. Blank, 3 negative and 1 positive in 1 hour. Total captures, 67 (58 positive, 9 negative).

t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charges.	Captures.	t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charges.	Captures.
16.8	12 *B*	47.0	0.112	1	1 P.	16.5	12 *B*	54.0	0.108	1	..
		9.6	.227	2	1 P.			10.2	.216	2	1 P.
		5.2	.349	3	..			5.4	.334	3	1 P.
	6 *B*	15.8	.342	3	1 P.		6 *B*	16.7	.328	3	..
		9.6	.455	4	..			10.0	.437	4	1 P.
								7.0	.543	5	1 P.
		15.6	.344	3	..						
		45.0	.228	2	1 N.		12 *B*	54.0	.108	1	..
		16.0	.340	3	1 P.			10.0	.219	2	1 P.
16.4		10.0	.444	4	1 P.			5.2	.344	3	1 P.
		6.9	.567	5	1 P.		6 *B*	17.0	.325	3	..
								10.0	.437	4	1 P.
	12 *B*	47.2	.112	1	..			7.2	.543	5	1 P.
		9.6	.226	2	1 P.						
		5.6	.330	3	1 P.	16.0	12 *B*	57.0	.107	1	..
	6 *B*	16.0	.338	3	..			10.0	.217	2	1 P.
		9.8	.446	4	1 P.						
16.4		6.9	.563	5	1 P.			58.0	.107	1	..
								16.0[1]	.000	0	1 N.
	12 *B*	48.0	.111	1	..			58.0	.107	1	1 P.
		9.9	.221	2	1 P.			10.0	.216	2	1 P.
		5.2	.344	3	1 P.			6.0	.306	3	1 P.
	6 *B*	16.0	.337	3	..		6 *B*	17.2	.322	3	..
		10.0	.438	4	1 P.			10.0	.434	4	1 P.
		7.0	.553	5	1 P.			7.2	.534	5	1 P.
16.2	12 *B*	50.0	.111	1	..		12 *B*	58.0	.106	1	..
		9.85	.223	2	1 P.			10.0	.216	2	1 P.
		5.5	.331	3	1 P.			5.8	.314	3	1 P.
	6 *B*	16.5	.332	3	..		6 *B*	17.2	.322	3	..
		10.0	.439	4	1 P.			57.5	.216	2	1 N.
		7.1	.550	5	1 P.			17.1	.322	3	1 P.
								10.0	.434	4	1 P.
	12 *B*	50.5	.110	1	1 P.			7.3	.533	5	1 P.
		10.0	.220	2	..						
		5.4	.336	3	1 P.	16.1	12 *B*	59.0	.106	1	..
	6 *B*	16.5	.332	3	..			10.2	.213	2	1 P.
		50.0	.221	2	1 N.			5.6	.326	3	1 P.
		16.5	.332	3	1 P.		6 *B*	17.3	.319	3	..
		10.0	.439	4	1 P.			10.2	.426	4	1 P.
		7.0	.555	5	1 P.			7.0	.543	5	1 P.

t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charges.	Captures.	t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charges.	Captures.
15.6	12 *B*	58.0	0.106	1	..	16.0	6 *B*	18.8	0.306	3	..
		10.2	.212	2	1 P.			61.0	.209	2	1 N.
		5.5	.323	3	1 P.			18.0	.313	3	1 P.
	6 *B*	18.4	.309	3	..			61.0	.209	2	1 N.
		10.7	.413	4	1 P.		12 *B*	60.0	.105	1	1 N.
		17.8	.314	3	1 N.			10.8	.205	2	1 P.
		10.6	.415	4	1 P.			5.6	.318	3	1 P.
		7.2	.531	5	1 P.		6 *B*	18.6	.306	3	..
								10.6	.415	4	1 P.
16.0	12 *B*	60.0	.105	1	..			18.8	.306	3	1 N.
		10.8	.205	2	1 P.			10.5	.417	4	1 P.
		5.8	.310	3	1 P.						

[1] Going downward (neutral).

not available at the time. However, by repeatedly washing the apparatus and etching with acid, it was found possible to reduce the rate of capture due to this induced radioactivity to one or two captures an hour. Then in all subsequent work the radium container was placed in the apparatus at the beginning of observations each day and withdrawn immediately after the close of the day's run, and in the interval between succeeding periods of observation the apparatus was washed and etched with acid if necessary. In order that the exact magnitude of this effect could be known for each series of observations, a "blank" determination of from thirty minutes' to an hour's duration was made each day at the beginning and end. In these "blank" determinations, the procedure was the same as in a regular run except that the radium door was not opened and thus the rate of capture due to induced radioactivity alone was determined. Blanks made in this way ranged from one to six captures an hour, being practically always greater at the end of the day.

Table I. gives a typical series of observations on a drop by this method. Column 1 gives the times under gravity: column 2, the times under the field (the notations 12*B* and 6*B* refer to the number of trays of batteries). It is to be noted that there is a gradual downward drift of the successive values of $[1 + (t_1/t_2)]1/N$ for the same charge on the drop. This is due to the slight change in the value of the weight of the drop and to the fact that mg/pe has been taken as constant. p is not absolutely constant as the voltage of each tray of 160 volts falls here at a rate of about 1.5 volts per hour. Hence the value of p is less at the end of the run than at the beginning, and if this correction were made, it would increase the various values. This is not at all necessary, however, as there can be no possible ambiguity in any instance in the choice of the number of charges on the drop.

It will be seen from Table I. that *out of 67 catches of ions there is not even one multiple charge. Such observations were taken in air with ten drops and a total of 624 captures with not a single multiple charge.* Table II. gives a summary of the data for air.

TABLE II.

Air.

Drop Number.	Positive Singles.	Negative Singles.	Multiple Captures.
1	11	3	0
2	58	9	0
3	39	8	0
4	49	29	0
5	79	39	0
6	37	15	0
7	13	13	0
8	21	11	0
9	17	13	0
10	101	59	0

V. CARBON DIOXIDE (CO_2).

This gas was made in a Kipp's generator using 50 per cent. nitric acid and marble (previously boiled out in nitric acid) washed by passing through a solution of sodium bicarbonate to eliminate acid spray, and passed through a drying system and dust filter before entering the apparatus. The carbon dioxide was passed into the oil drop chamber near the bottom under the condenser, and was always allowed to flow twice as long as was required to extinguish instantly a lighted match held an inch above the top of the vessel. Then the lid was put on air-tight and the vessel evacuated to a pressure of from 1 to 2 mm.; the final pressure of carbon dioxide was established by allowing the vessel to come into communication with a 20-liter bottle of the same air-free, dry carbon dioxide. This was considered to give an atmosphere sufficiently free from air to leave no doubt as to the effect of ionization of carbon dioxide.

With carbon dioxide, one double catch, or one set of two simultaneous single catches was observed; Table III. shows the observations on the droplet which caught this apparent double, while Table IV. gives the summary of the seven drops with which observations were conducted in carbon dioxide, *a total of 356 captures, all singles with the one exception noted above.*

VI. CARBON TETRACHLORIDE ($C.Cl_4$).

To obtain substances of higher molecular weight which could be used in the chamber of the oil-drop apparatus, it seemed best to choose liquids

TABLE III.

Drop No. 1 in Carbon Dioxide.

Pressure, 9.30 cm. Time of run, 48 minutes. Blank, 1 positive and 1 negative in 30 minutes. Total captures, 37 (28 positive, 8 negative, 1 apparent double positive).

t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charges.	Captures.
15.0	12 *B*	800	0.085	1	1 N.
		15.0[1]	.000	0	..
		800	.085	1	1 P.
		15.0[1]	.000	0	1 N.
		800	.085	1	1 P.
		7.4	.251	3	2 P.
	6 *B*	25.6	.263	3	..
		13.9	.346	4	1 P.
		10.0	.417	5	1 P.
		7.2	.513	6	1 P.
	12 *B*	Balanced	.085	1	..
		14.0	.171	2	1 P.
		7.4	.250	3	1 P.
	6 *B*	26.0	.261	3	..
		13.8	.345	4	1 P.
		10.0	.416	5	1 P.
14.6	12 *B*	Balanced	.083	1	..
		14.0	.170	2	1 P.
		6.8	.262	3	1 P.
	6 *B*	29.0	.251	3	..
		14.6	.333	4	1 P.
		28.7	.251	3	1 N.
		15.2	.326	4	1 P.
		10.0	.410	5	1 P.
14.6	12 *B*	Balanced	.083	1	..
		14.5	.168	2	1 P.
		7.6	.244	3	1 P.
	6 *B*	29.4	.249	3	..
		Balanced	.166	2	1 N.
		29.4	.249	3	1 P.
		15.0	.329	4	1 P.
		9.6	.420	5	1 P.
	12 *B*	Balanced	.083	1	..
		15.0[1]	.000	0	1 N.
		Balanced	.083	1	1 P.
		15.0[1]	.000	0	1 N.
		Balanced	.083	1	1 P.
		14.5	.167	2	1 P.
		7.4	.248	3	1 P.

t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charges.	Captures.
14.6	6 *B*	29.4	.250	3	..
		14.5	.334	4	1 P.
		10.0	.410	5	1 P.
	12 *B*	Balanced	.083	1	..
		15.0[1]	.000	0	1 N.
		Balanced	.083	1	..
		14.6[1]	.000	0	1 N.
		Balanced	.083	1	1 P.
		15.2	.163	2	1 P.
		7.4	.248	3	1 P.

[1] Going downward (neutral).

TABLE IV.

Carbon Dioxide.

Drop Number.	Positive Singles.	Negative Singles.	Multiple Captures.
1	28	8	1 double
2	53	20	0
3	77	23	0
4	13	2	0
5	37	13	0
6	4	0	0
7	64	13	0

of sufficiently high vapor tension to make it possible to work at the pressures which had been found to give the most suitable rate of capture. Of these liquids carbon tetrachloride appeared very promising on account of its indifference chemically, its stability, and its vapor pressure which at ordinary temperatures is about 10 cm. Three methods of filling the vessel with the vapor were employed, of which one was used with the first drop only and consisted in placing about five times as much of the liquid as would be required to fill the vessel as vapor at the working pressure and pumping until the saturation stage was just passed. As the time for attaining equilibrium was found to be excessive and as the vapors do not permit of proper operations of the pump, this method was abandoned for the more expeditious one of placing in a thin-walled glass bulb an amount of the liquid just sufficient to produce the proper pressure, suspending this in the ionizing chamber, and, after evacuating the latter, releasing the bulb by a device from without and thus breaking it and vaporizing the liquid. This method worked well, except that there were frequent failures of the glass bulb to break, but it was found that

TABLE V.

Drop No. 13 in Carbon Tetrachloride Vapor.

Pressure, 4.6 cms. Time of run, 1:40 hours. Blank, 4 catches in 40 minutes. Total captures 63 (49 positives, 14 negatives).

t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charges.	Captures.	t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charg s.	Captures.
14.2	10 *B*	42.4	0.133	1	..	10.9	6 *B*	13.1	0.305	3	1 P.
		8.6	.263	2	1 P.		4 *B*	50.4	.304	3	..
	5 *B*	56.0	.276	2	..			16.6	.413	4	1 P.
		16.0	.366	3	1 P.			10.2	.515	5	1 P.
	4 *B*	30.0	.358	3	..						
		14.6	.466	4	1 P.			50.4	.304	3	..
		9.3	.575	5	1 P.			16.6	.411	4	1 P.
								10.2	.515	5	1 P.
11.8	10 *B*	86.8	.114	1	..						
		9.0	.230	2	1 P.	10.6	10 *B*	Balanced	.105	1	..
	6 *B*	40.0	.215	2	..			10.2	.204	2	1 P.
		11.8	.324	3	1 P.		6 *B*	49.0	.203	2	..
		40.0	.215	2	1 N.			12.4	.309	3	1 P.
		12.2	.320	3	1 P.		4 *B*	54.0	.299	3	..
	4 *B*	40.0	.320	3	..			16.0	.416	4	1 P.
		16.4	.418	4	1 P.			10.8	.496	5	1 P.
		40.0	.320	3	1 N.						
		16.4	.416	4	1 P.	10.4	10 *B*	Balanced	.104	1	..
		10.0	.523	5	1 P.			10.4	.207	2	1 P.
									.201		
10.8	10 *B*	160	.107	1	..			Balanced	.104	1	..
		9.8	.210	2	1 P.			9.8	.207	2	1 P.
	6 *B*	40.4	.211	2	..		6 *B*	50.0	.201	2	..
		11.8	.319	3	1 P.			12.2	.309	3	1 P.
		42.0	.210	2	1 N.		4 *B*	65.0	.290	3	..
		12.6	.309	3	1 P.			18.4	.392	4	1 P.
		42.0	.210	2	1 N.			10.8	.491	5	1 P.
	10 *B*	220	.105	1	1 N.						
		9.6	.213	2	1 P.	10.2	10 *B*	[1]	.100	1	..
	6 *B*	48.8	.204	2	..			10.2	.202	2	1 P.
		12.4	.312	3	1 P.		6 *B*	51.0	.201	2	..
	4 *B*	47.1	.314	3	..			12.8	.302	3	1 P.
		17.8	.402	4	1 P.			50.0	.201	2	1 N.
		9.6	.532	5	1 P.		10 *B*	[1]	.100	1	1 N.
		17.2	.407	4	1 N.			10.4	.200	2	1 P.
		9.6	.532	5	1 P.			[1]	.100	1	1 N.
								10.2	.202	2	1 P.
10.9	10 *B*	250	.105	1	..		6 *B*	50.2	.201	2	..
		9.2	.217	2	1 P.			13.0	.296	3	1 P.
		250	.105	1	1 N.			51.0	.201	2	1 N.
		9.6	.213	2	1 P.			13.0	.296	3	1 P.
	6 *B*	44.6	.207	2	..			51.0	.201	2	1 N.

t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charges.	Captures.	t_1.	N.	t_2.	$\left(1+\frac{t_1}{t_2}\right)\frac{1}{N}$.	No. of Charges.	Captures.
10.2	6 *B*	13.0	0.296	3	1 P.	10.3	4 *B*	18.4	0.390	4	..
	4 *B*	18.8	.388	4	1 P.			10.6	.493	5	1 P.
		10.8	.491	5	1 P.						
						10.1	10 *B*	[1]	.100	1	..
10.3	10 *B*	[1]	.100	1	..			10.1[2]	.000	0	1 N.
		10.2	.200	2	1 P.			[1]	.100	1	1 P.
	6 *B*	50.0	.201	2	..			10.2	.200	2	1 P.
		12.0	.310	3	1 P.			[1]	.100	1	1 N.
		7.4	.399	4	1 P.			10.2	.200	2	1 P.

[1] Going downward very slowly.

[2] Going downward (uncharged).

the introduction of the same amount of liquid through the capillary of our atomizing arrangement worked satisfactorily and indeed yielded a steady state even more rapidly than did the plan just mentioned.

All droplets in carbon tetrachloride vapor exhibit a remarkable decrease in the time of fall under gravity. This is illustrated by drop No. 13 for which the data is reproduced in Table V. (see column headed t_1). This decrease is sufficiently explained by the solubility of the vapor in the watch oil used for the drops. That this is the correct explanation is shown by the fact that, after each atomization, there was an actual decrease of pressure due to the absorption of carbon tetrachloride vapor by the excess of oil in the large vessel.

Table VI. gives the summary of the fourteen drops with which observations were conducted in carbon tetrachloride vapor, totalling 457

TABLE VI.

Carbon Tetrachloride.

Drop Number.	Positive Singles.	Negative Singles.	Multiple Captures.
1	16	5	1 double
2	27	10	0
3	28	7	0
4	27	17	0
5	19	6	0
6	5	2	0
7	11	3	0
8	12	3	0
9	21	4	0
10	30	8	0
11	23	19	0
12	42	19	0
13	49	14	0
14	26	12	0

captures. *These are all singles with one exception which is again apparently a double.*

VII. METHYL IODIDE (CH_3I).

The methyl iodide was the ordinary commercial liquid not purified from the iodine which is always liberated when this substance is allowed to stand. The iodine, however, caused no disturbance because of any action on the metallic parts of the apparatus. In fact, this vapor acted much better than did the carbon tetrachloride in that there was less change in pressure and hence greater constancy in the speeds, although even here there is an unmistakable indication of the solubility of the vapor in the watch oil. The methyl iodide was introduced into the

TABLE VII.

Methyl Iodide.

Drop Number.	Positive Singles.	Negative Singles.	Multiple Captures.
1	15	3	0
2	43	9	0
3	38	22	0
4	22	7	0
5	19	7	0
6	92	32	0
7	48	13	0
8	42	23	1
9	27	16	0

ionizing chamber through the capillary tube of the atomizer after pumping down to 2 mm. Table VII. gives the summary of the nine drops used with the methyl iodide vapor, *showing a total of 478 single captures and one apparent double.*

VIII. MERCURY DIMETHYL $Hg(CH_3)_2$.

As Thomson's most striking results were obtained with mercury, this element was included in our list, the compound mercury dimethyl fortunately being sufficiently volatile to adapt itself to the procedure employed with the other liquids. The mercury dimethyl was prepared by the method of Frankland and Duppa,[1] and had the characteristic odor and gave nacreous crystals on treatment with alcoholic solution of iodine. It was introduced into the apparatus exactly as described under carbon tetrachloride and methyl iodide, *i.e.*, through the capillary tube of the atomizer. However, the vapor pressure did not rise above 2.5 cm., so that air was allowed to enter to bring the pressure up to 4.35 cm.: this means that the 24 liter vessel contained ten grams of mercury di-

[1] E. Frankland and B. F. Duppa, Jour. Chem. Soc., 16, 415 (1863).

methyl and one gram of air. On account of the extremely poisonous nature of the vapor, we made but one filling of the vessel, continuing the observations the second day until the blank reached a prohibitive value.

The 347 captures made with eleven drops are listed in Table VIII. *There are 346 "singles" and one apparent, double.*

TABLE VIII.

Mercury Dimethyl.

Drop Number.	Positive Singles.	Negative Singles.	Multiple Captures.
1	1	2	0
2	6	2	0
3	15	8	0
4	6	1	0
5	11	8	0
6	27	8	0
7	7	1	0
8	29	14	0
9	89	36	1 double
10	17	5	0
11	41	12	0

IX. DISCUSSION.

Over 2,900 captures of ions from atoms of atomic weight ranging from 1 to 200 have been observed. Of this number, there are only 5 captures which give any evidence of multiply charged atoms. These were all double positives. They could be attributed either to actual doubly-charged atoms, or to the simultaneous advent upon the drop of two singly-charged ions. That the latter is the correct conclusion seems to be fully warranted by the evidence. It has been shown that there were in reality two sources of ionization acting,—the α particle beam and the active deposit. While the latter was kept as low as possible, yet when both were acting, there would be a certain probability that captures from the two would occur simultaneously. In further support of this, some observations were taken when the background of ionization due to the active deposit was exceedingly high and then captures were observed following one another in such rapid succession that distinguishing them as separate events required the greatest vigilance on the part of the observer. Of the 5 captures which appeared to be doubles, 3 occurred just at the end of the day's run when the background had a value of 5 or 6 an hour and when the observer was not as alert as usual because of the long period of observation. It is not to be overlooked, too, that even at the low pressures used two separated molecules of the gas might be so situated as to be ionized by the same α particle and yet be thrown together upon the drop.

All of the heavier atoms used in this work were in chemical combination. This, according to Thomson[1] should rather favor the formation of multiply-charged atoms, yet we obtained no real indication of multiples.

Although all of the foregoing results are surprising, they are not at all irreconcilable with those obtained by Thomson. The time-integral of the force acting between any unit charge, moving through an atom, and any charge within the atom, varies inversely as the speed. The most rapid of the α particles used in these experiments were moving with a speed of 2×10^9 cm. per sec. If Thomson used potentials as high as 100,000 volts to produce his positive rays—and it is probable that this estimate is several times too high—the speed acquired by the hydrogen atom would have been but 5×10^8 cm. per sec. or one fourth that of the α rays. Since the time-integral of the ionizing force is directly proportional to the charge of the moving atom and inversely proportional to its speed, the 100,000-volt hydrogen atom would have twice as large a chance of producing ions as would the α ray. An oxygen atom which had acquired its speed in the same field as that acting upon the hydrogen atom would be moving but about one fourth as fast as the hydrogen, and hence would possess four times the ionizing power.

It is altogether conceivable, therefore, that slow positive rays might make multivalent ions when fast α rays would not. It is hoped to test this point by catching the positive residues of atoms formed by rays *at the very end of their range*, where they should be moving as slowly as the positive rays in Sir Joseph's experiments. No such attempts were made in the present work, in which the distance of the oil drops from the source of the rays amounted to but about one tenth of the range of the particles.

Again, it is to be remembered that the positive residues formed in these experiments practically never arise from nuclear or central impacts. For if the nucleus of the atom has a radius of the order 10^{-13} cm. while the atom itself has a radius of the order 10^{-8} cm., there would be ten billion atoms traversed by α rays to one nuclear impact, and the chance of catching on an oil drop a positive ion which arose from a nuclear impact would accordingly be altogether negligible.

There are, however, recent scintillation experiments of Rutherford's[2] which deal only with nuclear impacts, and *these seem to show that even such impacts upon atoms of nitrogen and oxygen produce primarily singly, not doubly or trebly charged atoms.*

[1] Sir J. J. Thomson, p. 54, Rays of Positive Electricity and their Application to Chemical Analysis, Longmans, Green, & Co.

[2] Phil. Mag., XXXVII., 578 (1919).

Since in Rutherford's experiments, the energy of impact of the α rays upon the nucleus is much greater than the energy of impact of positive rays can ever be, his failure to obtain multivalent nitrogen and oxygen atoms is certainly surprising, and this, taken in connection with the present experiments, shows clearly that the conditions under which multiply charged atoms may be formed are quite limited. Such atoms are surely not formed in the positive ray work primarily, if at all, by nuclear impacts; for the numbers in which they appear seem to be much greater than the possible number of such impacts. And further, according to Rutherford such impacts produce in general singles, not multiples, in these gases. Again, the present experiments show that multivalent ions are not formed by the non-nuclear impacts of high speed α rays. It is probable that they are formed only in a narrow range of speeds just above that which makes interpenetration of the atoms possible.

Only in the cases of carbon and helium does Rutherford infer multiply charged atoms as a result of his nuclear impacts and the inference is here drawn from negative evidence rather than positive, viz., from the failure to obtain scintillations at the ranges at which such scintillations should appear, if singly charged carbon and helium atoms were formed. Our own experiments, while they gave in carbon dioxide only singly charged positive residues, might conceivably give doubly charged ones with helium. It is hoped in the near future to subject this gas to tests of the sort herein reported.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO,
July 30, 1919.

THE THEORY OF RECEIVERS FOR SOUND IN WATER.

By H. A. WILSON.

SYNOPSIS.

Sound receivers in water; mathematical theory. (1) *Single receivers.* After proving that a receiver must have a small volume to be efficient, the author considers in detail the case of a small spherical receiver connected to the ear by a cylindrical tube. Its resonance frequency is independent of the size of the tube and depends only on the elasticity of the receiver and its effective mass. The sharpness of resonance, however, is greater for larger tubes and for smaller surfaces. The energy transmitted for a given pitch is a maximum for a certain tube size; and while it is independent of the surface at the resonance frequency, for lower pitches the intensity increases with the surface. It is shown that a resonating receiver with a properly chosen tube may concentrate into the tube energy from an area about 850 times the cross section of the tube. The theory agrees with experimental facts and leads to practical suggestions for the design of efficient receivers. (2) *Two receivers as used for direction finding.* A pair of receivers mounted on a horizontal rod which may be rotated about a vertical axis is an efficient device for getting the direction of a source of sound since it makes use of the binaural effect. The equations for the energy in each tube and for the difference in phase are derived. Unless the separation is very small, the conditions for correct direction finding are always satisfied. The best distance apart is half the wave-length in water for the resonance frequency of the receivers. (3) *Multiple receivers.* If n receivers are each connected by a tube with a cross section α to a tube whose cross section is $n\alpha$, and if the lengths are such that all the sounds meet in phase, practically all the sound may be concentrated in a single tube. Equations are derived for the case of symmetrically arranged receivers, all equidistant from the source of sound. Except at the resonance frequency, the multiple receivers should be, in practice, more sensitive than any single receiver. (4) *Lines of receivers.* By a special compensating device the sound from such a set may be concentrated in a single tube. Equations are derived for the energy transmitted in this case. (5) *Receivers distributed over large areas.* With flexible diaphrams, complete transmission may be secured if the ratio of the tube section to the diaphram area is properly chosen. With stiff diaphrams this is approximately true only for the resonance frequency of the receivers. Finally, the conditions for complete transmission for a given frequency are determined for the case of small receivers mounted in a plane in front of a totally reflecting parallel surface.

THIS paper contains an account of the theory of the action of " Broca tubes " and similar receivers for sound in water when used either singly or in combinations of any number.

1. SINGLE RECEIVERS.

The original Broca tube was simply a stethoscope which could be immersed in water. It consisted of a flat circular metal box one of the

circular sides of which was made of a thin metal plate. A tube fixed into the center of the opposite side, led to the ears of the observer. Such an apparatus is shown in Fig. 1. When immersed in water and acted on by sound waves, the flexible diaphragm is made to vibrate by the pressure variations in the water and these vibrations are communicated to the air in the box and produce sound waves in the tube.

The box and diaphragm may be replaced by a rubber tube closed at one end as shown in Fig. 2, or by a spherical rubber ball, Fig. 3.

If the linear dimensions of the receiver are small compared with the

Fig. 1.

Fig. 2.

Fig. 3.

wave-length of the sound in air, then we can suppose that the pressure variation in the receiver is the same throughout its volume and is given by the equation

$$\delta P = -\gamma P \delta V / V,$$

where P denotes the pressure of the undisturbed air, δP the pressure change due to the sound, V the volume of the receiver, δV the volume variation due to the sound and γ the ratio of the specific heat of the air at constant pressure to that at constant volume.

If we suppose that $\delta V_1 = B\epsilon^{ipt}$ where δV_1 denotes the volume variation due to the motion of the diaphragm, B denotes the maximum value of δV_1, t denotes the time and $p = 2\pi n$, where n is the frequency, then we can easily calculate the amount of sound energy which passes up the tube leading from the receiver.

Let the displacement of the air in the tube from its undisturbed position be denoted by ξ and let

$$\xi = A\epsilon^{i(pt - k_1 x)}, \tag{1}$$

where x denotes the distance along the tube from the receiver and $k_1 = p/v_1$ where v_1 is the velocity of sound in air. Then at the junction between the receiver and the tube, the air pressure in the receiver must be equal to the air pressure in the tube so that at $x = 0$

$$\delta P = -\gamma P \delta V / V = -\gamma P d\xi / dx. \tag{2}$$

Also we have, if α denotes the cross section of the tube

$$\delta V = \delta V_1 + \alpha \underset{x=0}{\xi}, \tag{3}$$

because the total variation of the volume of the air in the receiver is equal to the variation due to the motion of the diaphragm plus that due to the motion of the air at the end of the tube. The equations (1), (2), and (3) give

$$A = \frac{B\epsilon^{i\theta}}{\sqrt{\alpha^2 + k_1^2 V^2}} \qquad \text{where} \qquad \tan\theta = -k_1 V/\alpha.$$

The flow of energy along the tube therefore varies as

$$\alpha A^2 = \frac{B^2\alpha}{\alpha^2 + k_1^2 V^2}.$$

For a given value of V this is a maximum when $\alpha = k_1 V$ but for a given value of α it is greater, the smaller the value of V.

It is clear that V should be made small; a conclusion amply confirmed by experience. Provided $k_1 V$ is small compared with α it may be neglected and we get $A = B/\alpha$ which shows that then the volume of air leaving the receiver along the tube at $x = 0$ is equal to the diminution of the volume due to the motion of the walls of the receiver. In most cases which occur in practice $k_1 V$ is smaller than α. For example, a standard $\frac{1}{4}$ inch rubber receiver has an interior volume of about one c.c. so that for a wave-length $\lambda = 60$ cms. $k_1 V$ is about 0.1 and α is about 0.3 sq.cm. Also the effective volume of a tubular rubber receiver is probably much less than the actual volume because a large part of its volume variation is due to variations in the length of the receiver rather than to changes in its diameter.

The theory of sound receivers is greatly simplified when $k_1 V$ is small compared with α so in the rest of this paper it will be assumed that this is the case and the diminution of the volume of the receiver will be put equal to the volume of air expelled from it as though the air in the receiver were an incompressible fluid.

The variation of the volume of the receiver is, of course, due to the pressure variations in the surrounding water. These pressure variations are partly due to the incident sound and partly to the sound emitted into the water by the receiver itself. In the case of a spherical receiver, the sound emitted can be easily calculated but in the case of cylindrical and circular receivers the calculation is complicated. It is easy to see that the action of a receiver depends mainly on its volume variation and that its shape makes little difference except in so far as it influences the volume variation. In what follows, the receiver will be supposed

spherical, as in Fig. 3, so that the results obtained should be exact for spherical receivers and will only be approximately true for receivers of any other form.

An elastic sphere of radius a immersed in water and vibrating radially, will emit spherical sound waves in which the pressure variation is inversely as the distance r from the center of the sphere. If the radius of the sphere is very small compared with the wave-length of the sound in water, then

$$\delta P = -\frac{Ek^2a^2\xi\epsilon^{-ikr}}{r},$$

where δP denotes the pressure variation in the water due to the emitted spherical waves, E the bulk modulus of elasticity for water, $k = 2\pi/\lambda$ where λ is the wave-length of the sound in water and ξ denotes the radial displacement of the surface of the sphere.

This result may be obtained as follows: Let the velocity potential of the emitted waves be given by

$$\phi = Br^{-1}\epsilon^{i(pt-kr)}$$

and let

$$\xi = D\epsilon^{ipt};$$

then

$$\delta P = -\rho\frac{d\phi}{dt} = -\rho\frac{Bip}{r}\epsilon^{i(pt-kr)}.$$

Also

$$\dot{\xi} = \frac{d\phi}{dr}(r = a) = -B\left(\frac{1}{a^2} + \frac{ik}{a}\right)\epsilon^{i(pt-kr)}$$

$$= Dip\epsilon^{ipt}.$$

Hence

$$\delta P = -\frac{\rho p^2a^2\xi\epsilon^{-ikr}\cdot\epsilon^{ika}}{r(1 + ika)}.$$

Also $E = \rho v^2$ and when ka is very small $\epsilon^{ika} = 1 + ika$ so that we get the equation given above for δP.

We can now go on to calculate the intensity of the sound produced in the tube leading from the receiver in terms of the intensity of the incident sound in the water.

The vibration of the surface of the spherical receiver is due to the variation of the outside water pressure minus the variation of the inside air pressure. Also we may suppose that the motion of the surface is resisted by an elastic restoring force proportional to the displacement and by a viscous resistance proportional to the velocity.

Let the pressure variation in the water due to the incident sound be denoted by $A\epsilon^{ipt}$. The total pressure variation in the water at $r = a$

will then be $A\epsilon^{ipt} - Ek^2a\xi\epsilon^{-ika}$. Let the potential of the sound in the tube be given by $\phi = G\epsilon^{i(pt-k_1x)}$, where x is the distance from the receiver measured along the tube.

Then the air pressure in the receiver where $x = 0$ is equal to

$$-\rho_1\frac{d\phi}{dt} = -\rho_1 Gip\epsilon^{ipt},$$

where ρ_1 = density of air.

Let m denote the mass of the spherical receiver, a its radius when undisturbed by sound, ξ the radial displacement of the surface due to sound, μ the restoring force per unit radial displacement due to the elasticity of the sphere, R the internal viscous resistance to the motion per unit radial velocity. Then we have

$$m\ddot{\xi} + R\dot{\xi} + \mu\xi = -4\pi a^2(A\epsilon^{ipt} - Ek^2a\xi\epsilon^{-ika} + Gip\rho_1\epsilon^{ipt}). \quad (4)$$

We have also at $x = 0$ assuming that the air inside the receiver behaves like an incompressible fluid

$$\alpha\frac{d\phi}{dx} = -4\pi a^2\dot{\xi},$$

so that putting $4\pi a^2 = s$ we get

$$\xi = \frac{\alpha G}{sv_1}\epsilon^{ipt}.$$

Substituting this for ξ in (4) and putting $\epsilon^{+ika} = 1 + ika$ we get

$$G = \frac{-s^2v_1\alpha^{-1}A\epsilon^{-i\theta}}{\sqrt{(\mu - p^2(m + s\rho a))^2 + p^2\left(R + \frac{\rho s^2p^2}{4\pi v} + \frac{\rho_1 s^2 v_1}{\alpha}\right)^2}}, \quad (5)$$

where

$$\tan\theta = \frac{p\left(R + \frac{\rho s^2p^2}{4\pi} + \frac{\rho_1 s^2 v_1}{\alpha}\right)}{\mu - p^2(m + s\rho a)}.$$

If the elastic sphere were vibrating in a vacuum under the action of a radial force $-A\epsilon^{ipt}$ per unit area we should have

$$m\ddot{\xi} + R\dot{\xi} + \mu\xi = -sA\epsilon^{ipt},$$

so that if $\xi = B\epsilon^{ipt}$ we get

$$B = \frac{-sA\epsilon^{-i\theta}}{\sqrt{(\mu - mp^2)^2 + R^2p^2}},$$

where

$$\tan\theta = \frac{Rp}{\mu - mp^2}.$$

Comparing this with (5), we see that the mass of the sphere forming the receiver in water may be regarded as increased by $s\rho a$ and the viscous

resistance to its motion by $\rho s^2 p^2/4\pi v$ due to the water and by $\rho_1 s^2 v_1/\alpha$ due to the air inside it. The additional viscous resistance is thus proportional to the square of the surface of the receiver so that very small receivers will be highly resonant provided R is small. For materials such as rubber and brass, experiments indicate that R is negligible when p is greater than about 100. R may therefore be put equal to zero.

For small receivers for which the total viscous resistance is small the expression for G is a maximum approximately when $\mu - p^2(m + s\rho a) = 0$. Let p_1 denote the value of p for which this is the case so that when $p = p_1$ and $R = 0$

$$G = \frac{iv_1 A}{p_1\left(\frac{\alpha\rho p^2}{4\pi v} + \rho_1 v_1\right)}.$$

In this case G has a phase 90° behind that of the incident sound and is independent of the physical properties of the sphere, except as regards p_1 but it depends on α the cross section of the tube leading from the receiver.

The energy E of the sound going up the tube, in unit time, is given by

$$E = \tfrac{1}{2}\rho_1 v_1 k_1^2 G^2 \alpha = \frac{1}{2}\frac{\rho_1}{v_1} p^2 G^2 \alpha.$$

Hence

$$E = \frac{\tfrac{1}{2}\rho_1 v_1 p^2 s^4 \alpha^{-1} A^2}{(\mu - p^2(m + s\rho a))^2 + p^2\left(R + \frac{s^2\rho_1 v_1}{\alpha} + \frac{s^2\rho p^2}{4\pi v}\right)^2}.$$

Hence when $p = p_1$ and $R = 0$

$$E = \frac{\tfrac{1}{2}\rho_1 v_1 \alpha A^2}{\left(\frac{\alpha\rho p_1^2}{4\pi v} + \rho_1 v_1\right)^2}. \tag{6}$$

This shows that E is a maximum with respect to α when

$$\rho_1 v_1 = \alpha\rho p_1^2/4\pi v \qquad \text{or} \qquad \alpha = \rho_1 v_1 \lambda_1^2/\pi\rho v,$$

where λ_1 is the wave-length in water for the frequency $p_1/2\pi$. In this case

$$E = \frac{1}{8}\frac{\alpha A^2}{\rho_1 v_1} = \frac{A^2\lambda_1^2}{8\pi\rho v}.$$

The amount of sound energy transmitted up the tube from the receiver can be conveniently expressed in terms of the area of wave front in the incident sound through which an equal amount of sound energy flows. If β denotes this area then $E = A^2\beta/2\rho v$ so that when $p = p_1$ and α has the best value

$$\frac{1}{2}\frac{A^2\beta}{\rho v} = \frac{A^2\lambda_1^2}{8\pi\rho v}$$

or

$$\beta = \frac{\lambda_1^2}{4\pi}.$$

Also when $p = p_1$ and α has the best value

$$E = \frac{1}{8}\frac{\alpha A^2}{\rho_1 v_1} = \frac{1}{2}\frac{\beta A^2}{\rho v},$$

so that

$$\frac{\beta}{\alpha} = \frac{1}{4}\frac{\rho v}{\rho_1 v_1} = 853.$$

Thus it appears that a receiver at its resonance frequency, when the cross section of the tube leading from it has the best value for this frequency, can concentrate into the tube the sound from an area 853 times that of the cross section of the tube.

In the expression found for E for any value of α if we put $M = m + s\rho a$ and $\alpha_1 = 4\pi v\rho_1 v_1/\rho p_1^2$ we get

$$E = \frac{\frac{1}{2}(\rho_1 v_1)^{-1}\alpha A^2}{\frac{\alpha^2 M^2}{(\rho_1 v_1)^2 s^4}\left\{\frac{p_1^2 - p^2}{p}\right\}^2 + \left\{1 + \frac{\alpha}{\alpha_1}\frac{p^2}{p_1^2}\right\}^2}.$$

This expression is a maximum with respect to α when

$$\alpha = \left\{\frac{M^2}{s^4(\rho_1 v_1)^2}\left(\frac{p_1^2 - p^2}{p}\right)^2 + \frac{p^4}{\alpha_1^2 p_1^4}\right\}^{-\frac{1}{2}},$$

which gives $\alpha = \alpha_1$ when $p = p_1$.

As an example suppose $a = 2$ cms., $M = 134$ grams, $p_1 = 5{,}000$,

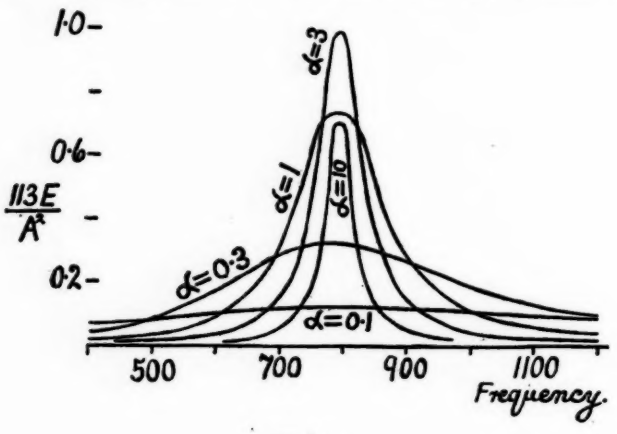

Fig. 4.

$R = 0$, $\alpha_1 = 3$ sq. cms. Fig. 4 shows the values of $113E/A^2$ in this case for five values of α, viz., 10, 3, 1, 0.3, and 0.1. It will be seen that increasing α makes the receiver more sharply resonant but diminishes

the energy received both above and below the resonance frequency. This suggests a method of finding the resonance frequency of a receiver. If the receiver is connected to a wide tube of section much greater than α_1 it will be very insensitive except for frequencies near to its resonance frequency. The resonance frequency is not changed appreciably by varying α.

It is clear that if it is desired to make a receiver sensitive over a wide range of frequencies then the tube should be made with a section smaller than α_1. As the result of very many experiments with many types of receivers the following facts have been established when listening to a noise in the water like that from a boat or submarine:

1. The sound heard always has a more or less definite pitch characteristic of the receiver used.

2. Receivers having equal tubes and equal pitches give sounds of about equal intensity.

3. With receivers having equal tubes those of higher pitch give less intensity.

All the receivers for which the above results are true have small interior volumes.

Receivers with large volumes give less intensity than those with small volumes.

These results are in accordance with the theory. The sound from the boat is a noise having all frequencies between certain limits in its spectrum. The loudness falls off as the frequency increases. The receivers respond best to the frequencies near to p_1 for which the expression for E has its maximum value. This explains fact (1). Equation (6) shows that the sound energy obtained at the resonance frequency depends only on p_1 and α and that for a given value of α it decreases as p_1 increases. This agrees with facts (2) and (3).

It appears therefore that in the design of receivers, there are four things to which special attention should be given:

(1) Resonance frequency,
(2) Cross section of tube,
(3) Interior volume,
(4) Surface.

The resonance frequency determines the character of the sound received. Experience seems to show that it should be as high as is compatible with sufficient intensity. The best resonance frequency seems to be about 800 per sec. for many purposes.

The cross section of the tube should be equal to or rather less than $\alpha_1 = \rho_1 v_1 \lambda_1^2 / \pi \rho v$ where λ_1 is the wave-length in the water at the resonance

frequency. This gives about 3 sq. cms. for a frequency of 800 per sec. for which $\lambda_1 = 180$ cms. The interior volume of the receiver should be considerably less than $\alpha\lambda'/2\pi$ where λ' is the wave-length in air corresponding to the resonance frequency. If $\alpha = 3$ sq. cms. and $\lambda' = 43$ cms., then V should be much less than 20 c.c.; $V = 2$ c.c. would be quite small enough.

These conclusions apply to single receivers which when in use are not near to other receivers. The case of multiple receivers is considered in section (3) of this paper. As to surface, this has no influence at the resonance frequency but below the resonance frequency increasing the surface rapidly increases the intensity of the sound received. Receivers with small surfaces are sharply tuned while receivers with a large surface are sensitive over a wide range of frequencies.

When $p = p_1$ and $\alpha = \alpha_1$ we have seen that $E = \frac{1}{8}\alpha_1 A^2/\rho_1 v_1$ where A denotes the amplitude of the pressure variation in the water due to the incident sound. If Q denotes the amplitude of the pressure variation in the air in the tube leading from the receiver then $E = \frac{1}{2}\alpha_1 Q^2/\rho_1 v_1$ so that $Q = A/2$. Thus at the resonance frequency, when α has the best value for this frequency, the pressure variation in the air is one half the pressure variation in the water due to the incident sound.

2. Two Receivers, One Connected to Each Ear.

An important case is that in which two receivers are used, one connected to each ear by tubes of equal length. The two receivers are fixed on a horizontal line, in the water, at a constant distance apart. The two receivers can be rotated about a vertical axis bisecting the line joining them. The observer rotates the apparatus until the sound appears to come from a point directly in front of his head and then the source lies in the plane perpendicular to the line joining the two receivers and passing through its middle point. If the apparatus is turned slightly from this position the sound will appear to the observer to move towards the receiver which is moving nearer to the source. In this way the direction of the source in the horizontal plane can be accurately determined. This arrangement was proposed by the writer early in June, 1917, was developed by the General Electric Company and has been extensively employed during the war by the United States Navy.

The theory of this device can be worked out in the same way as that of a single receiver. The pressure variation on each receiver now includes a term due to the sound emitted by the other receiver. It will be supposed that the two receivers are exactly similar spheres and that their interior volume is small.

Let the distance between the two receivers be denoted by $AB = l$ and let the angle between the line joining them AB and the direction of the source of sound be denoted by ϕ. Let the pressure variation due to the incident sound at A be given by

$$A\epsilon^{i(pt-\frac{1}{2}kl\cos\phi)}$$

and that at B by

$$A\epsilon^{i(pt+\frac{1}{2}kl\cos\phi)}.$$

Let $\xi = P\epsilon^{ipt}$ denote the displacement of the surface of the sphere at A and $\eta = Q\epsilon^{ipt}$ denote the displacement of the surface of the sphere at B. Also let the velocity potential of the sound in the tube leading from A be denoted by

$$G\epsilon^{i(pt-k_1x)}$$

and that of the sound in the other tube by

$$H\epsilon^{i(pt-k_1x)}$$

then, as in the case of the single receiver, we have

$$\frac{P}{s}(\mu - mp^2 + Rip) - p^2aP\epsilon^{-ika} = -A\epsilon^{-i\Delta} + Q\frac{p^2a^2}{l}\epsilon^{-ikl} - \rho_1 Gip,$$

$$\frac{Q}{s}(\mu - mp^2 + Rip) - p^2aQ\epsilon^{-ika} = -A\epsilon^{i\Delta} + P\frac{p^2a^2}{l}\epsilon^{-ikl} - \rho_1 Hip,$$

where Δ has been written for $\frac{1}{2}kl\cos\phi$ and the density of water has been put equal to unity. Also we have

$$\xi = \frac{\alpha G\epsilon^{ipt}}{sv_1} \quad \text{and} \quad \eta = \frac{\alpha H}{sv_1}\epsilon^{ipt}$$

so that $P = \alpha G/sv_1$ and $Q = \alpha H/sv_1$ as in the case of a single receiver. Substituting these values of P and Q and solving the two equations for G and H we obtain:

$$H = -A\left\{\frac{\cos\Delta}{K-J} + \frac{i\sin\Delta}{K+J}\right\},$$

$$G = -A\left\{\frac{\cos\Delta}{K-J} - \frac{i\sin\Delta}{K+J}\right\},$$

where

$$K = \frac{\alpha}{s^2v_1}(\mu - mp^2 + Rip - p^2as\epsilon^{-ika}) + \rho_1 ip$$

and

$$J = \frac{\alpha p^2a^2\epsilon^{-ikl}}{sv_1l}.$$

Let

$$M = m + as + (sa^2\cos kl)/l$$

and

$$S = R + \frac{p^2a^2s}{v} + \frac{s^2v_1\rho_1}{\alpha} + \frac{spa^2}{l}\sin kl,$$

so that

$$K - J = \frac{\alpha}{s^2v_1}(\mu - Mp^2 + ipS).$$

Also let

$$N = m + as - (sa^2\cos kl)/l$$

and

$$T = R + \frac{p^2a^2s}{v} + \frac{s^2\rho_1v_1}{\alpha} - \frac{spa^2\sin kl}{l},$$

so that

$$K + J = \frac{\alpha}{s^2v_1}(\mu - Np^2 + ipT).$$

Now let

$$\frac{\alpha}{s^2v_1}(\mu - Mp^2) = h \qquad \text{and let} \qquad \frac{\alpha}{s^2v_1}pS = g.$$

Also let

$$\frac{\alpha}{s^2v_1}(\mu - Np^2) = u \qquad \text{and let} \qquad \frac{\alpha}{s^2v_1}pT = w.$$

Hence

$$H = -A\left\{\frac{\cos\Delta}{h+ig} + \frac{i\sin\Delta}{u+iw}\right\},$$

$$G = -A\left\{\frac{\cos\Delta}{h+ig} - \frac{i\sin\Delta}{u+iw}\right\}.$$

These equations give

$$H \doteq -A\epsilon^{i\psi}\sqrt{\frac{\cos^2\Delta}{h^2+g^2} + \frac{\sin^2\Delta}{u^2+w^2} - \frac{\sin 2\Delta\cdot(gu-hw)}{(h^2+g^2)(u^2+w^2)}},$$

$$G = -A\epsilon^{i\psi'}\sqrt{\frac{\cos^2\Delta}{h^2+g^2} + \frac{\sin^2\Delta}{u^2+w^2} + \frac{\sin 2\Delta\cdot(gu-hw)}{(h^2+g^2)(u^2+w^2)}},$$

where

$$\tan\psi = \frac{(u^2+w^2)g\cos\Delta - (h^2+|g^2)u\sin\Delta}{(u^2+w^2)h\cos\Delta + (h^2+g^2)w\sin\Delta}$$

and

$$\tan\psi' = \frac{(u^2+w^2)g\cos\Delta + (h^2+g^2)u\sin\Delta}{(u^2+w^2)h\cos\Delta - (h^2+g^2)w\sin\Delta}.$$

Hence

$$\tan(\psi'-\psi) = \frac{2(hu+gw)\tan\Delta}{u^2+w^2-(h^2+g^2)\tan^2\Delta}.$$

Thus there are two resonance frequencies approximately given by $h = 0$ and $u = 0$ but unless l is very small they almost coincide.

The determination of the direction of the incident sound requires that when $\Delta = 0$ then $\psi' - \psi = 0$ and that, for small values of Δ,

$\psi' - \psi$ shall be of the same sign as Δ. The formula for $\tan(\psi' - \psi)$ shows that when $\Delta = 0$ then $\psi' - \psi = 0$ in all cases, whatever the distance between the receivers. When Δ is small, so that $(h^2 + g^2)\tan^2\Delta$ can be neglected, $\psi' - \psi$ will have the same sign as Δ provided $hu + gw$ is positive. Now $hu + gw$ is equal to

$$\left(\frac{\alpha}{s^2 v_1}\right)^2 (\mu - Mp^2)(\mu - Np^2) + \left(\frac{\alpha}{s^2 v_1}\right)^2 p^2 ST,$$

which might be negative if p were between $\sqrt{\mu/M}$ and $\sqrt{\mu/N}$. When l is not very small M and N are nearly equal so that when p is between $\sqrt{\mu/M}$ and $\sqrt{\mu/N}$ then both $\mu - Mp^2$ and $\mu - Np^2$ are very small so that $hu + gw$ can not be negative unless ST is very small which is never the case. Consequently unless l is very small, which is never the case in practice, the conditions for giving correct direction are always satisfied.

When Δ is not very small the sound will all appear to come from the same direction provided $\psi' - \psi = 2\Delta$ for all frequencies audible. This will be the case if $h = u$ and $g = w$, for then

$$\tan(\psi' - \psi) = \frac{2\tan\Delta}{1 - \tan^2\Delta} = \tan 2\Delta.$$

If l is large compared with the radius a of the receivers then the two resonance frequencies are near together so that for frequencies near them, both h and u will be small. In this case approximately

$$\tan(\psi' - \psi) = \frac{2gw\tan\Delta}{w^2 - g^2\tan^2\Delta},$$

which is equal to $\tan 2\Delta$ if $g = w$. Now

$$g - w = \frac{\alpha}{s^2 v_1} p(S - T) = \frac{\alpha p^2}{2\pi v_1 l}\sin kl,$$

which is independent of the size of the receivers. If $\sin kl = 0$ or $kl = \pi$ then $g = w$ so that if $l = \lambda/2$ where λ is the wave-length in the water for the resonance frequency, we shall have $\psi' - \psi = 2\Delta$ approximately for frequencies near the resonance frequency and the greater part of the sound heard will then appear to all come from the same direction whatever the value of Δ.

It appears that the best value of l is equal to half the wave-length in the water for the resonance frequency of the receivers.

3. The Theory of Multiple Receivers.

In order to obtain more sound than can be got from a single receiver a number of receivers can be used connected by tubes to a single tube leading to the ear. The use of such multiple receivers was proposed in

June, 1917, by Professor Zeleny and the writer, who worked out the theory according to which receivers should be connected together and also showed experimentally that more sound could be obtained from multiple receivers than from single ones.

Multiple receivers designed according to the principles worked out by Professor Zeleny and the writer have since been employed extensively in anti-submarine work by the United States Navy.

Suppose a number of tubes with sections $\alpha_1\alpha_2\alpha_3\cdots\alpha_n$ are joined into a single tube of section γ. Let the velocity potential in the mth tube be

$$A_m\epsilon^{i(pt-kx)} + B_m\epsilon^{i(pt+kx)}$$

and in the single tube

$$C\epsilon^{i(pt-kx)} + D\epsilon^{i(pt+kx)}.$$

Here x is the distance measured along the tubes and we shall suppose that x has the same value x_1 at the junction in all the potentials.

Then since the pressure at the junction has the same value in all the tubes, we must have

$$A_m\epsilon^{-ikx_1} + B_m\epsilon^{ikx_1}$$

equal to a constant P say, whatever the value of m. Also the total flow of air into the junction along the n tubes must be equal to the flow out along the single tube so that

$$\sum_1^n \alpha_m(A_m\epsilon^{-ikx_1} - B_m\epsilon^{ikx_1}) = \gamma(C\epsilon^{-ikx_1} - D\epsilon^{ikx_1}).$$

Putting $B_m\epsilon^{ikx_1} = P - A_m\epsilon^{-ikx_1}$ this gives

$$\Sigma\alpha_m(2A_m\epsilon^{-ikx_1} - P) = \gamma(2C\epsilon^{-ikx_1} - P).$$

Hence we see that if $\gamma = \Sigma\alpha_m$ then

$$C = (\Sigma\alpha_m A_m)/\gamma$$

and in the same way if $\gamma = \Sigma\alpha_m$ then

$$D = (\Sigma\alpha_m B_m)/\gamma.$$

Suppose a multiple receiver is made up of n similar receivers each connected to a tube of section α and let the potential in the mth tube be $A_m\epsilon^{i(pt-kx)} + B_m\epsilon^{i(pt+kx)}$. Let the n tubes be joined together, in any way, into a single tube of section γ in which the potential is $C\epsilon^{i(pt-kx)}$. Then if at all junctions between two or more tubes the total cross section is conserved we shall have $\gamma = n\alpha$,

$$C = \frac{\Sigma\alpha_m A_m}{\gamma} = \frac{\Sigma A_m}{n},$$

and

$$\Sigma B_m = 0.$$

Also if the A's are all equal then $C = A$. In this case all the sound comes out along the single tube.[1]

The A's will all be equal if all the receivers produce equal amplitudes of vibration and if the phases of the vibrations are all the same at places where x has the same value. This requires that if the phase at the mth receiver is θ_m and x_m denotes the value of x at this receiver then $\theta_m + kx_m$ must have the same value for all the receivers and if all the receivers vibrate with equal phases then x_m must have the same value for all the receivers.

It appears that the sounds from any number of equal receivers can all be combined into a single tube if when two or more tubes are joined the area of cross section is conserved and if the paths from each receiver to the final tube are such that $\theta_m + kx_m$ has the same value for all the receivers.

Since sharp bends in a tube may produce some reflection, the tubes should also be as nearly straight as possible. If desired, the sound can be concentrated into a narrower tube by means of a cone provided the cross section is not changed much in a length equal to the wave-length of the sound.

The theory of a multiple receiver designed in accordance with the above principles will now be considered. Let there be n equal receivers each connected to a tube of section α and let the n tubes be combined into a single tube of section $n\alpha$. To simplify the theory it will be supposed that the paths from each receiver to the final tube are all of the same length. The receivers will also be supposed to be arranged in a symmetrical way so that each one bears the same relation to the others; for example this would be the case if the receivers were arranged on a circle at equidistant points.

Each receiver will be supposed to be an elastic sphere of small internal volume and the notation used will be the same as in the previous sections of this paper.

Let ξ_m denote the radial displacement of the surface of the mth sphere and then we have

$$m\ddot{\xi}_m + R\dot{\xi}_m + \mu\xi_m = -s\delta P, \qquad (1)$$

where δP denotes the total pressure variation on the surface of the sphere. This pressure variation δP is made up of (1) the pressure due to the source, (2) the pressure due to the sound emitted by the mth sphere, (3) that due to the sound from the other spheres, (4) that due to the sound in the air tube. (1) will be denoted by $S\epsilon^{i(pt+\theta_m)}$, (2) is equal

[1] See Theory of Sound, Rayleigh, Vol. II., p. 63.

to $-\rho a p^2 \xi_m \epsilon^{-ika}$, (3) is equal to $-\rho a^2 p^2 \Sigma \frac{\xi \epsilon^{-ikr}}{r}$, in which there is one term for each of the other spheres ξ being the radial displacement and r the distance from the mth sphere measured between centers. (4) is equal to $\rho_1 ip(A_m + B_m)\epsilon^{ipt}$. (2) and (3) can be combined into $-\rho a^2 p^2 \sum_1^n \frac{\xi \epsilon^{-ikr}}{r}$, in which for the mth sphere $r = a$ and $\xi = \xi_m$.

Hence equation (1) becomes

$$m\ddot{\xi}_m + R\dot{\xi}_m + \mu\xi_m = -s\left\{ S\epsilon^{i(pt+\theta_m)} - \rho p^2 a^2 \sum_1^n \frac{\xi \epsilon^{-ikr}}{r} + \rho_1 ip(A_m + B_m)\epsilon^{ipt} \right\}. \quad (2)$$

We have

$$s\dot{\xi}_m = \alpha i k_1 (A_m - B_m)\epsilon^{ipt},$$

so that

$$\xi_m = \frac{\alpha}{s v_1}(A_m - B_m)\epsilon^{ipt}.$$

Putting this value of ξ_m in (2) and adding up the n equations like (2) we get, after some reduction remembering that $\Sigma B_m = 0$ and that owing to the assumed symmetry

$$\sum_1^n \sum_1^n \frac{\xi \epsilon^{-ikr}}{r} = \sum_1^n \xi_m \sum_1^n \frac{\epsilon^{-ikr}}{r},$$

$$\Sigma A_m = \frac{-s^2 v_1 n S \Sigma \epsilon^{i\theta_m}}{\alpha\left(\mu - mp^2 + Rip + \frac{is^2\rho_1 p v_1}{\alpha} - \frac{s^2\rho p^2}{4\pi}\sum_1^n \frac{\epsilon^{-ikr}}{r}\right)}.$$

In what follows it will be supposed that all the receivers are equally distant from the source in the water so that

$$\sum_1^n \epsilon^{i\theta_m} = n.$$

If the potential in the tube leading from the multiple receiver is $C\epsilon^{i(pt-k_1 x)}$ then as we have seen

$$C = \frac{\Sigma A_m}{n}.$$

The amount of sound energy E passing along this tube in unit time is $\frac{1}{2}\rho_1 v_1 k_1^2 C^2 \alpha n$, so that

$$E = \frac{\rho_1 v_1 s^4 p^2 S^2 n}{2\alpha\left\{\left(\mu - mp^2 - \frac{s^2\rho p^2}{4\pi}\Sigma\frac{\cos kr}{r}\right)^2 + p^2\left(R + \frac{s^2\rho_1 v_1}{\alpha} + \frac{s^2\rho p}{4\pi}\Sigma\frac{\sin kr}{r}\right)^2\right\}}$$

when $n = 1$ this reduces to the expression previously found for a single

receiver since

$$\frac{\cos ka}{a} = \frac{1}{a} \qquad \text{and} \qquad \frac{\sin ka}{a} = k.$$

In the case of rubber or brass receivers it is probable that we may put $R = 0$. The expression for E shows that, for a certain value of p, E has a maximum value. Usually this maximum will occur near to the value of p given by

$$\mu - mp^2 - \frac{s^2\rho p^2}{4\pi}\Sigma\frac{\cos kr}{r} = 0,$$

which will be denoted by p_1. For a single receiver p_1 is given by

$$\mu - mp^2 - s\rho a p^2 = 0$$

so that we see that p_1 for a multiple receiver is smaller than for a single receiver. This result agrees with observations. When $p = p_1$

$$E = \frac{\rho_1 v_1 Sn}{2\alpha\left(\frac{\rho_1 v_1}{\alpha} + \frac{\rho p}{4\pi}\Sigma\frac{\sin kr}{r}\right)^2}.$$

This is a maximum with respect to α when

$$\alpha = \frac{4\pi\rho_1 v_1}{\rho p_1 \Sigma\frac{\sin kr}{r}}.$$

When α has this value which we will denote by α_1 and $p = p_1$ then

$$E = \frac{S^2 n\alpha}{8\rho_1 v_1} = \frac{\pi S^2 n}{2\rho p_1 \Sigma\frac{\sin kr}{r}}.$$

If all the receivers are rather near together so that kr is small then

$$\Sigma\frac{\sin kr}{r} = nk,$$

approximately and

$$\Sigma\frac{\cos kr}{r} = \frac{1}{a} + \frac{n-1}{\bar{r}},$$

where $(\bar{r})^{-1}$ denotes the mean value of r^{-1}. Hence in this case when $R = 0$

$$E = \frac{\frac{1}{2}\rho_1 v_1 s^4 p^2 S^2 n\alpha}{\alpha^2\left\{\mu - p^2\left[m + s\rho a\left(1 + \frac{(n-1)a}{\bar{r}}\right)\right]\right\}^2 + p^2\left\{s^2\rho_1 v_1 + \frac{s^2\rho p^2 n\alpha}{4\pi v}\right\}^2}.$$

This expression is of the same form as that previously found for a single receiver so that it appears that a multiple receiver in which the

distances between the receivers are small compared with the wave-length in the water, can theoretically be replaced by an equivalent single receiver.

Let $\alpha' s' \mu' m' p_1'$ denote the values of these quantities for the equivalent single receiver. Then we must have $\alpha' = n\alpha$, $\mu' = \mu s'^2/ns^2$ and $p_1' = p_1$ for if these equations are true the expression for E, just given, reduces to that for a single receiver. These three equations contain only three unknown quantities because μ m and s are determined in terms of the elastic properties and density of the sphere when its dimensions are known.

At the resonance frequency $p = p_1$ and

$$E = \frac{\frac{1}{2}\rho_1 v_1 S^2 n\alpha}{\left(\rho_1 v_1 + \frac{\rho p_1^2 n\alpha}{4\pi v}\right)^2}.$$

This shows that at the resonance frequency a multiple receiver, in which the receivers are rather near together, can be replaced by any single receiver for which $\alpha' = n\alpha$ and which has the same resonance frequency as the multiple receiver. If in addition $ns^2\mu' = s'^2\mu$ the single receiver and the multiple receiver will be equivalent at all frequencies.

If κ denotes the bulk modulus of elasticity and ν the rigidity modulus of the material of which the receivers are made then

$$\mu = \frac{48\pi\kappa\nu\tau}{(\kappa + \frac{4}{3}\nu)},$$

where τ is the thickness of the hollow sphere. Let

$$\frac{48\pi\kappa\nu}{\kappa + \frac{4}{3}\nu} = C$$

so that $\mu = C\tau$. Let δ denote the density of the material of the receivers. Then the equation $p_1' = p_1$ gives

$$\frac{\mu'}{m' + s'\rho a'} = \frac{\mu}{m + s\rho a(1 + (n-1)a/\bar{r})},$$

where

$$m' = 4\pi a'^2 \delta' \tau'; \qquad m = 4\pi a^2 \delta\tau$$

$$\mu' = C'\tau'; \qquad \mu = C\tau.$$

The equation $ns^2\mu' = \mu s'^2$ gives

$$\tau' = \frac{a'^4 \tau C}{na^4 C'},$$

so that we get

$$\frac{C'}{C} = \frac{a'^3\delta'}{a^2\{a'(\delta + a/\tau + a^2(n-1)/\tau\bar{r} - na^2/\tau\}}. \qquad (3)$$

We see from this that a' must be greater than

$$na^2/(\tau\delta + a + a^2(n-1)/\bar{r}).$$

If a' is taken big enough to satisfy this condition then if a material can be found for which C'/δ' has the value given by (3) it will be possible to construct the equivalent single receiver out of it.

The equivalent single receiver can not be made of the same material as the multiple receiver unless the equation in a'

$$a^2\{a'(\delta + a/\tau + a^2(n-1)/\tau\bar{r}) - na^2/\tau\} = a'^3\delta$$

has positive real roots which is not the case in practical cases.

For example if $n = 8$, $a = 1$ cm., $\tau = 0.1$ cm., $\bar{r} = 14$ cms., $\delta = 1$ this equation becomes

$$a'^3 = 16a' - 80,$$

which has no positive real roots so that in this case a single equivalent receiver can not be made out of the same material as the multiple receiver.

But if we take $a' = 7$ cms. we get $C'/C = 10{\cdot}7\delta'$, so that a single equivalent receiver of radius 7 cms. could be made out of a material for which $C'/\delta' = 10{\cdot}7C$. If $\delta' = 1$ this gives $\tau' = 2.8$ cms. If a material were available for which $C' = 1{,}000C$ and $\delta' = 10$ then we have

$$100\{a'(1 + 10 + 5) - 80\} = a'^3$$

which has a positive root $a' = 5.08$ cms. approximately. This gives $\tau' = 0.00832$ cm. A receiver with a radius of 5 cms. would have too large an internal volume for this to be neglected but a solid core could be put inside it so as to make its effective volume small. A receiver of 5 cms. radius and thickness only 0.00832 cm. would be too thin to be useful for practical purposes.

If the receivers in the multiple receiver are made of rubber then C is very small compared to its value for other substances so that it is then practically impossible to make a single equivalent receiver because when C'/C is large the thickness of the single receiver becomes very small unless its radius is very large. This follows from $\tau' = \tau a'^4 C/na^4C'$.

It appears that single equivalent receivers can not be made for practical purposes. This conclusion agrees with experience, for a great many attempts were made to make single receivers equivalent to a multiple rubber receiver without any success.

If for a multiple receiver $n = 10$, $a = 3$ cms., $m = 100$ grams, $\bar{r} = 15$ cms. and the resonance frequency $N_1 = 328$, the equation for E/S^2 when α has the best value at the resonance frequency becomes

$$\frac{E}{S^2} = \frac{0.213}{51.5\left(\frac{1-f^2}{f}\right)^2 + (1+f^2)^2},$$

where $f = N/N_1$.

For a single receiver equal to each of the 10 receivers in the case just given we get $N_1 = 478$ and when α again has the best value for this frequency

$$\frac{E}{S^2} = \frac{0.100}{536\left(\frac{1-f^2}{f}\right)^2 + (1+f^2)^2}.$$

Fig. 5 shows the values of E/S^2 given by these equations for different values of N.

It appears that the multiple receiver is more sensitive than the single

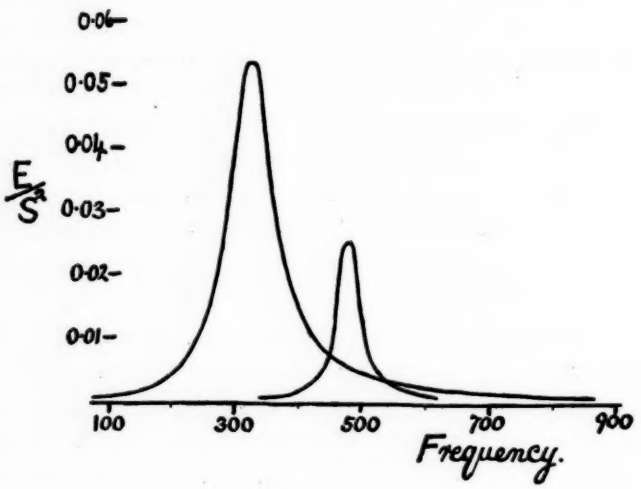

Fig. 5.

one except very near the resonance frequency of the single receiver. The single receiver is also of higher pitch and is more sharply resonant.

As one more example take a multiple receiver for which $n = 8$, $a = 1$ cm., $\delta = 1$, $\tau = 0.1$ cm., $p_1 = 5{,}000$, $\alpha = 3/8$ sq. cm. These values give

$$113\frac{E}{S^2} = \frac{4}{32.2\left(\frac{1-f^2}{f}\right)^2 + (1+f^2)^2}.$$

Also take a single receiver for which $a' = 2$, $\alpha' = 3$, $\delta' = 1$, $\tau' = 2/3$ cm. and $p_1' = 5{,}000$. This single receiver has the same resonance frequency and the same sized tube as the multiple receiver and it is

made of the same material but it does not satisfy the equation $ns^2\mu' = \mu s'^2$. For this receiver

$$113\frac{E}{S^2} = \frac{4}{354\left(\frac{1-f^2}{f}\right)^2 + (1+f^2)^2}.$$

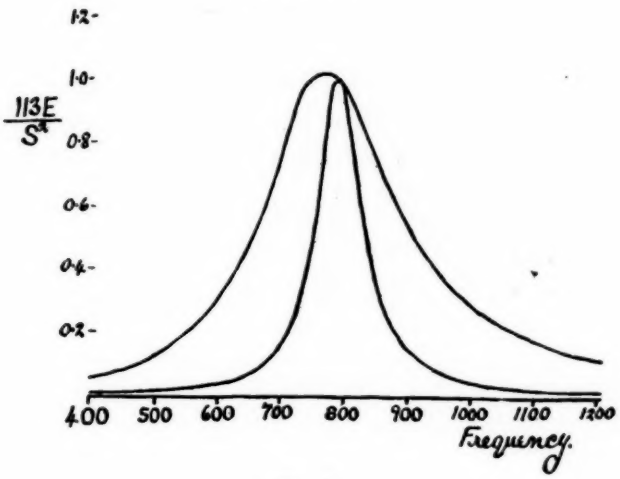

Fig. 6.

The relation between $113E/S^2$ and N for these two receivers is shown in Fig. 6. It appears that the multiple receiver is more sensitive except at the frequency $N_1 = p_1/2\pi$ where they are equally sensitive.

4. Lines of Receivers.

The theory of a number of receivers, either single or multiple, arranged with uniform spacing along a straight line will now be considered. This arrangement was proposed by Professor Max Mason early in June, 1917, and has been extensively employed by the United States Navy during the war.

If the distance from any receiver to the next one in the line is d and if the angle between the line and the direction of the incident sound is ϕ then the distance the sound has to travel in the water increases, as we go along the line from one receiver to the next one, by $d \cos \phi$. If v is the velocity of sound in water the time difference from one receiver to the next one is therefore $(d \cos \phi)/v$. The receivers are connected by tubes to a single tube leading to the ear. In order that the sound from all the receivers may arrive at the ear at the same instant it is necessary that the tubes be of different lengths. The difference of length for two adjacent receivers must be $(v_1 d \cos \phi)/v$, where v_1 is the velocity of sound in air. Thus as ϕ varies the differences between the lengths of suc-

cessive tubes must be varied in order that the sounds from all the receivers may always arrive together at the ear so as to produce the maximum possible intensity. Instruments for producing this variation were devised by Professor Mason and are called compensators.

It will be assumed here that the tube lengths are such that the sounds from all the receivers arrive together at the ear. In this case for a very long line the angle of incidence of the external sound makes no difference so long as the intensity of the external sound at each receiver remains unchanged. When two or more tubes are connected together the area of cross section must be conserved as in the multiple receivers considered in the previous section.

The theory of the multiple receiver can be applied to the case of a long line of receivers equally spaced for this theory applies to receivers equally spaced round the circumference of a circle and if we suppose the diameter of the circle to be indefinitely increased the sound energy from each receiver will be nearly the same as from each receiver in a long line.

In this case we have

$$\Sigma \frac{\sin kr}{r} = k + 2\left\{\frac{\sin kd}{d} + \frac{\sin 2kd}{2d} + \cdots\right\}$$

$$= \pi/d \text{ when } d < \lambda.$$

$$= k \text{ when } d \text{ is very large.}$$

Also

$$\Sigma \frac{\cos kr}{r} = \frac{1}{a} + 2\left\{\frac{\cos kd}{d} + \frac{\cos 2kd}{2d} + \cdots\right\}$$

$$= \frac{1}{a} - \frac{1}{d}\log\left(4\sin^2\left(\frac{kd}{2}\right)\right).$$

Hence the sound energy from a length l of the line when d is less than λ will be given by

$$E = \frac{\rho_1 v_1 s^4 p^2 S^2 l}{2\alpha d\left\{\left[\mu - mp^2 - s\rho a p^2\left(1 - \frac{a}{d}\log\left(4\sin^2\left(\frac{kd}{2}\right)\right)\right)\right]^2 + p^2\left[\frac{s^2\rho_1 v_1}{\alpha} + \frac{s^2\rho p}{4d}\right]^2\right\}}.$$

At the resonance frequency this becomes

$$E = \frac{\rho_1 v_1 \alpha S^2 l}{2d(\rho_1 v_1 + \rho p_1 \alpha/4d)^2}.$$

This is a maximum with respect to α/d when

$$\alpha/d = 4\rho_1 v_1/\rho p_1 = 27/N_1,$$

where

$$N_1 = p_1/2\pi.$$

When α/d has this value then

$$E = \frac{S^2 l}{2\rho p_1} = \frac{S^2 l}{4\pi\rho N_1}.$$

If β denotes the area of the incident wave fronts through which an amount of energy equal to E flows then at the resonance frequency when $\alpha/d = 4\rho_1 v_1/\rho p_1$

$$E = \frac{1}{2}\frac{S^2\beta}{\rho v} = \frac{S^2 l}{4\pi\rho N_1},$$

so that

$$\frac{\beta}{l} = \frac{v}{2\pi N_1} = \frac{\lambda_1}{2\pi},$$

where λ_1 is the wave-length in the water for the resonance frequency.

We may observe that if, at the resonance frequency $\sin(kd/2) = \frac{1}{2}$ or $d = \lambda_1/6$ then $\log(4\sin^2(kd/2)) = 0$, so that $p_1^2 = \mu/(m + s\rho a)$ as for a single receiver.

If d is less than $\lambda_1/6$ the resonance frequency will be lower than for a single receiver. If $d = \lambda_1/6$ then the equation $\alpha/d = 27/N_1$ gives

$$\alpha = \frac{9\lambda_1}{2N_1} = \frac{9}{2}\frac{\lambda_1^2}{v} = \frac{\lambda_1^2}{32000}.$$

5. Receivers Distributed over Large Areas.

Instruments have been proposed in which a large number of sound receivers was to be uniformly distributed over a considerable area on the side of a ship. If the area covered has linear dimensions greater than the wave length of the sound in the water then the sound energy transmitted per unit area may be calculated approximately by assuming that the area covered with receivers is an infinite plane on which plane waves are incident. It will be convenient to begin by considering a simple ideal case.

Consider an infinite plane completely covered with thin flexible diaphragms each of area β. Suppose that behind each diaphragm there

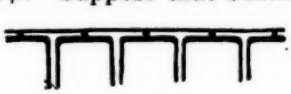

Fig. 7.

is a small enclosed space of volume V from which a tube of section α leads. This arrangement is shown in Fig. 7. Suppose that the space on the right of the plane is filled with water and that the cavities and tubes are filled with air and that a train of sound waves in the water is travelling towards the plane.

Let the velocity potential of the incident sound be given by

$$\phi = \phi_1 \epsilon^{i(ax+by+ct)},$$

where x denotes the distance from the infinite plane.[1] Let the potential of the reflected waves in the water be given by

$$\phi' = \phi_2 \epsilon^{i(-ax+by+ct)}$$

and let the potential in the tubes leading from the receivers be denoted by

$$\phi'' = \phi_3 \epsilon^{i(a_1 x+by+ct)}.$$

In the tubes $(-x)$ denotes the distance from the receivers measured along the tubes.

Let us suppose that the diaphragms are massless and perfectly flexible so that the air pressure in the cavities must be equal to the pressure in the water. In this case we have at $x = 0$

$$-\rho_1 \frac{d\phi''}{dt} = -\rho\left(\frac{d\phi}{dt} + \frac{d\phi'}{dt}\right)$$

where ρ_1 = density of air and ρ = density of water. Also, if the volume V is very small, we may put the flow of air along the tubes at $x = 0$ equal to the flow of water over the plane $x = 0$.

Hence at $x = 0$

$$\beta\left(\frac{d\phi}{dx} + \frac{d\phi'}{dx}\right) = \alpha \frac{d\phi''}{dx}.$$

These equations give

$$\rho_1 \phi_3 = \rho(\phi_1 + \phi_2)$$

and

$$\alpha a_1 \phi_3 = \beta a(\phi_1 - \phi_2),$$

so that

$$1 - \left(\frac{\phi_2}{\phi_1}\right)^2 = \frac{4\frac{\rho\alpha a_1}{\rho_1\beta a}}{\left\{1 + \frac{\rho\alpha a_1}{\rho_1\beta a}\right\}^2}.$$

Now $(\phi_2/\phi_1)^2$ is the fraction of the incident sound energy which is reflected so that $1 - (\phi_2/\phi_1)^2$ is equal to the fraction of the incident sound energy which is transmitted from the water into the air in the tubes. If $\rho\alpha a_1/\rho_1\beta a = 1$ then $1 - (\phi_2/\phi_1)^2 = 1$ so that all the energy is transmitted and there is no reflected sound.

If θ denotes the angle of incidence then $a = (2\pi \cos\theta)/\lambda$ and $a_1 = (2\pi \cos\theta_1)/\lambda_1$ where λ is the wave-length in water and λ_1 that in air. Also θ_1 is given by $v/v_1 = \sin\theta/\sin\theta_1$ where v denotes the velocity of sound in water and v_1 that in air. Hence for complete transmission

[1] See Theory of Sound, Rayleigh, Vol. II., p. 79.

we must have

$$\alpha = \beta \frac{\rho_1 v_1 \cos\theta}{\rho v \cos\theta_1}.$$

When $\theta = 0$ this gives $\alpha = \beta/3390$ approximately and when $\theta = \pi/2$ it gives $\alpha = 0$.

It appears that, theoretically, complete transmission can be secured for any given angle of incidence by giving the proper value to α/β.

The difference between the properties of water and air can be compensated by concentrating the sound into a much smaller area in the air by means of the tubes from each receiver.

If the tubes and enclosed spaced are omitted, we get $\alpha = \beta$ so that

$$1 - (\phi_2/\phi_1)^2 = \frac{4\frac{\rho a_1}{\rho_1 a}}{\left\{1 + \frac{\rho a_1}{\rho_1 a}\right\}^2},$$

which is equal to the fraction of the sound transmitted from water to air when there is nothing in between them. This fraction is only about 0.12 per cent. at normal incidence.

Now suppose that the diaphragms are elastic so that if ξ denotes the mean displacement of a diaphragm in the x direction then at $x = 0$

$$\frac{d\phi}{dx} + \frac{d\phi'}{dx} = \xi \tag{1}$$

and

$$\delta p + \rho\left(\frac{d\phi}{dt} + \frac{d\phi'}{dt}\right) = \mu\xi + m\dot{\xi}, \tag{2}$$

where δp denotes the variation of the air pressure in the cavity and μ denotes the pressure required to produce unit value of ξ and m denotes the effective mass of the diaphragms.

Let η denote the air displacement in the tubes at $x = 0$ so that

$$\delta p = -\gamma p \delta V/V = -\gamma p(\xi\beta - \alpha\eta), \tag{3}$$

where γ is the ratio of the specific heats of air. Also at $x = 0$

$$-\rho_1 \frac{d\phi''}{dt} = \delta p \tag{4}$$

and

$$\frac{d\phi''}{dx} = \dot{\eta}. \tag{5}$$

These five equations, after eliminating ξ, η, δp and ϕ_3 give

$$1 - \left(\frac{\phi_2}{\phi_1}\right)^2 = \frac{4\alpha\beta\rho\rho_1 a a_1}{\{\alpha a_1\rho + \beta a\rho_1 + aV(\mu - mc^2)/v_1^2\}^2 + \{V\rho c^2/v_1^2 - \alpha a a_1(\mu - mc^2)/c^2\}^2}.$$

If $\mu = 0$, $m = 0$ and $V = 0$ this reduces to the expression previously obtained.

In the case of normal incidence we have $a = c/v$ and $a_1 = c/v_1$ so that

$$1 - \left(\frac{\phi_2}{\phi_1}\right)^2 = \frac{4\alpha\beta\rho\rho_1 v v_1}{\{\alpha\rho v + \beta\rho_1 v_1 + V(\mu - mc^2)/v_1\}^2 + \{V\rho c v/v_1 - \alpha(\mu - mc^2)/c\}^2}.$$

If V is so small that the terms containing it can be neglected this expression has a maximum value with respect to c when $c = \sqrt{\mu/m}$ which corresponds to the natural frequency of the diaphragms when vibrating freely. When $c = \sqrt{\mu/m}$ and V can be neglected the expression is again the same as that previously obtained. It appears that if $\alpha = \beta\rho_1 v_1/\rho v$ then at the resonance frequency, when V is very small, all the sound will be transmitted but at all other frequencies some will be reflected.

It is not possible in practice to make V so small that it can be neglected so that complete transmission can not be secured. If V is not small enough to be neglected then the best value of α is greater than that given by $\alpha = \beta\rho_1 v_1/\rho v$.

As an example suppose $\beta = 50$ sq. cms., $m = 2$ gms., $\alpha = 1$ sq. cm., $V = 2$ c.c., $\mu = 10^7$. The following values of $1 - (\phi_2/\phi_1)^2$ are then obtained.

c	$1-(\phi_2/\phi_1)^2$
500	0.0560
1000	0.0577
5000	0.0534
10000	0.0419

If α is equal to 0.1 sq. cm. instead of 1 sq. cm. we get:

c	$1-(\phi_2/\phi_1)^2$
338	0.416
1000	0.344
5000	0.0589
10000	0.0162

Thus the smaller value of α gives better results at low frequencies but worse at high frequencies.

Instead of a large area covered with flexible diaphragms an area over

Fig. 8.

which small sound receivers are distributed uniformly may be used. Each receiver is connected to a tube and the plane containing the receivers is at a certain distance l from the side of the ship. This arrangement is shown in Fig. 8. AB is the plane containing the receivers and CD

the side of the ship. The side of the ship reflects almost perfectly the sound which is incident upon it. The reflecting surface is approximately the surface of the air inside the wall of the ship. To increase the effective distance between the layer of receivers and the air surface metal plates may be put in between. Such metal plates are nearly equivalent acoustically to an equal mass of water. It will be supposed that the length l is the thickness of water acoustically equivalent to the actual structure between the receivers and the reflecting air surface.

If the distance between adjacent receivers is small compared with the wave-length of the sound in the water and if the radius of a receiver is not very small compared with the distance between adjacent receivers then it will be approximately correct to suppose that when plane sound waves are incident the reflected waves are also plane. We have to consider five trains of plane waves. The incident waves, the reflected waves to the right of AB and the two sets of waves between AB and CD and the waves in the air tubes leading from the receivers.

Let the potential of the incident sound waves be given by

$$\phi_1' = \phi_1 \epsilon^{i(ax+by+ct)},$$

where x denotes the distance from the plane AB containing the receivers. Let the potential of the reflected sound to the right of AB be given by

$$\phi_2' = \phi_2 \epsilon^{i(-ax+by+ct)}.$$

Let the potentials of the two sets of waves in the water between AB and CD be given by

$$\phi_3' = \phi_3 \epsilon^{i(ax+by+ct)},$$

$$\phi_4' = \phi_4 \epsilon^{i(-ax+by+ct)}.$$

Let the potential of the sound in the air tubes be given by

$$\phi_5' = \phi_5 \epsilon^{i(a_1 x+by+ct)},$$

where $(-x)$ is the distance from the receivers measured along the tubes.

If we suppose that the volume of the receivers can be neglected then at $x = 0$ we may put the total flow of water towards the plane $x = 0$ equal to the flow of air into the air tubes from the receivers. Hence

$$-\frac{d\phi_1'}{dx} - \frac{d\phi_2'}{dx} + \frac{d\phi_3'}{dx} + \frac{d\phi_4'}{dx} = -f\frac{d\phi_5'}{dx},$$

where f denotes the ratio of the total cross section of the tubes to the area covered with receivers. Therefore

$$-\phi_1 + \phi_2 + \phi_3 - \phi_4 = -\phi_5 a_1 f/a. \qquad (1)$$

Also, since the pressure in the water at $x = 0$ has only one average value

at any instant we get

$$\phi_1 + \phi_2 = \phi_3 + \phi_4. \tag{2}$$

It will be supposed that the diminution of the volume of the receivers is proportional to the difference between the pressure in the water and the pressure in the air so that

$$\phi_1 + \phi_2 = \phi_5\left(\frac{\rho_1}{\rho} + \frac{i\mu a_1}{\rho c^2}\right), \tag{3}$$

where ρ denotes the density of water ρ_1 that of air and μ is an elastic constant for the receivers equal to the pressure difference required to produce unit air displacement in the tubes at $x = 0$.

Also at $x = -l$ the total pressure variation is zero so that

$$\phi_4 = -\phi_3\epsilon^{-2ial}. \tag{4}$$

Eliminating ϕ_3, ϕ_4 and ϕ_5 we obtain

$$\phi_2 = -\phi_1 \frac{\left\{\frac{\rho_1 a}{\rho a_1} + \frac{\mu a}{\rho c^2}\tan al + i\left(\left(f - \frac{\rho_1 a}{\rho a_1}\right)\tan al + \frac{\mu a}{\rho c^2}\right)\right\}}{\left\{\frac{\rho_1 a}{\rho a_1} - \frac{\mu a}{\rho c^2}\tan al + i\left(\left(f + \frac{\rho_1 a}{\rho a_1}\right)\tan al + \frac{\mu a}{\rho c^2}\right)\right\}}.$$

If $\phi_2 = 0$ there is no reflected sound so that all the incident sound is transmitted into the air tubes. This will be the case provided

$$\rho_1 a/\rho a_1 + (\mu a \tan al)/\rho c^2 = 0$$

and

$$(f - \rho_1 a/\rho a_1)\tan al + \mu a/\rho c^2 = 0,$$

or if $\tan al = -\rho_1 c^2/\mu a_1$ and $f = \rho_1 a/\rho a_1 + a a_1 \mu^2/\rho\rho_1 c^2$.

The following table gives the values of l and f for which there is complete transmission for several values of μ and n for the case of normal incidence.

$\mu = 3.5 \times 10^5$.			$\mu = 7 \times 10^5$.			$\mu = 14 \times 10^5$.		
n.	l.	f.	n.	l.	f.	n.	l.	f.
1,500	34.4	0.00053	3,000	17.25	0.00053	3,000	19.75	0.00118
1,000	56.0	0.00080	1,500	39.4	0.00121	1,500	43.0	0.00379
			1,000	62.6	0.00227	1,000	67.0	0.00815

Here n = frequency and l is in cms.

If $\mu = 0$ so that the receivers are perfectly flexible then for complete transmission $\tan al = -\infty$ so that $al = \pi/2$, or $l = \lambda/4 \cos\theta$ where θ is the angle of incidence, and $f = \rho_1 a/\rho a_1$ which is equal to $\rho_1 v_1/\rho v$ in the case of normal incidence. Calculating $1 - (\phi_2/\phi_1)^2$ we get it equal to

$$\frac{4f\rho a_1/\rho_1 a}{\left\{1 + \left(\frac{\mu a_1}{\rho_1 c^2}\right)^2\right\}\cot^2 al + \frac{2\mu\rho a_1^2 f}{\rho_1^2 a c^2}\cot al + \left(\frac{\mu a_1}{\rho_1 c^2}\right)^2 + \left\{1 + f\frac{\rho a_1}{\rho_1 a}\right\}^2},$$

which in the case of normal incidence reduces to

$$\frac{4\gamma}{\left\{1+\left(\frac{\alpha}{n}\right)^2\right\}\cot^2\phi+\frac{2\alpha\gamma}{n}\cot\phi+\left(\frac{\alpha}{n}\right)^2+(1+\gamma)^2},$$

where $\gamma = f\rho v/\rho_1 v_1$, $\alpha = \mu/2\pi\rho_1 v_1$, $\phi = 2\pi nl/v$. This expression is equal to unity when $\tan\phi = -n/\alpha$ and $\gamma = 1 + \alpha^2/n^2$, which are, of course, equivalent in the case of normal incidence, to the two conditions previously obtained.

Fig. 9 shows the percentage of sound transmitted into the air tubes

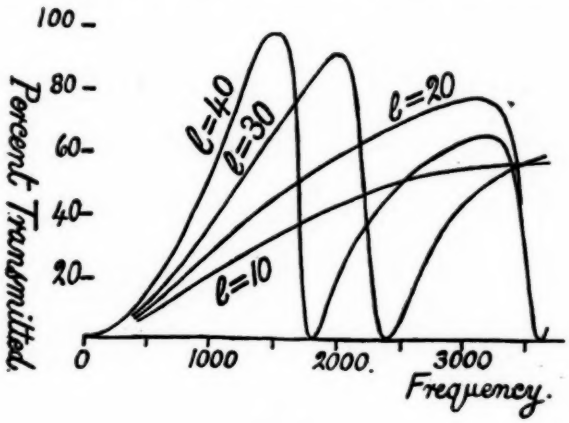

Fig. 9.

for different values of the frequency n and l in the case when $\mu = 7 \times 10^5$, $f = 0.00136$ and $\theta = 0$.

It will be seen that the maximum transmission occurs for a value of n which is about 0.85 of that for which the transmission is zero. Even with small values of l there is considerable transmission at high frequencies.

In conclusion I wish to express my thanks to Professor O. D. Kellogg for valuable help in connection with several parts of this paper. The greater part of the work described above was done at the Naval Experimental Station, New London, Conn., in connection with experimental investigations on anti-submarine devices.

RICE INSTITUTE,
HOUSTON, TEX.,
May, 1919.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

EINSTEIN'S LAW OF GRAVITATION.[1]

BY J. S. AMES.

THE by-laws of our society make it one of the duties of its president to deliver an address before its members. This fact renders it necessary for the president to select a subject; and this year the selection is to a certain degree forced by the public press. When a daily newspaper considers Einstein's work on gravitation a topic of sufficiently general interest to devote to it valuable space and cable funds, surely here is justification for my selection of this as the subject of my presidential address.

Einstein's original memoirs upon Gravitation appeared in the years 1916 to 18; and there are two excellent papers in English expounding and explaining his method, one by Professor de Sitter of Leyden and one by Professor Eddington of Cambridge. While Einstein's work may be known to many of you either in its original form or in one of the two papers mentioned, I fear that the attention of most of us was first directed seriously to the matter by the articles in the newspapers to which I have referred. I confess that I was one of those who had postponed any serious study of the subject, until its immense importance was borne in upon me by the results of the recent eclipse expedition. I have all the enthusiasm of the discoverer of a new land, and feel compelled to describe to you what I have learned.

Albert Einstein, although now a resident of Berlin and holder of a research professorship of the Kaiser Wilhelm Institut, is legally a Swiss. He is forty-five years old and was for some time a professor in the Zurich Technical School, and later in the University of Prague. He is a man of liberal tendencies, and apparently one whom any of us would be glad to welcome for personal reasons in our international meetings of the future. He protested against the famous manifesto of the German professors in 1914 and was one of the eager supporters of the German Republic when it arose from the wreck of the Empire.

But, in presenting the subject of Einstein's study of the law of gravitation, I must begin many years ago. In the treatment of Maxwell's equations of the electromagnetic field, several investigators realized the importance of

[1] Presidential address delivered at the St. Louis meeting of the Physical Society, December 30, 1919.

deducing the form of the equations when applied to a system moving with a uniform velocity. One object of such an investigation would be to determine such a set of transformation formulae as would leave the mathematical form of the equations unaltered. The necessary relations between the new space-coördinates, those applying to the moving system, and the original set were of course obvious; and elementary methods led to the deduction of a new variable which should replace the time coördinate. This step was taken by Lorentz and also, I believe, by Larmor and by Voigt. The mathematical deductions and applications in the hands of these men were extremely beautiful, and are probably well known to you all.

Lorentz' paper on this subject appeared in the Proceedings of the Amsterdam Academy in 1904. In the following year there was published in the Annalen der Physik a paper by Einstein, written without any knowledge of the work of Lorentz, in which he arrived at the same transformation equations as did the latter, but with an entirely different and fundamentally new interpretation. Einstein called attention in his paper to the lack of definiteness in the concepts of time and space, as ordinarily stated and used. He analyzed clearly the definitions and postulates which were necessary before one could speak with exactness of a length or of an interval of time. He disposed forever of the propriety of speaking of the "true" length of a rod or of the "true" duration of time, showing, in fact, that the numerical values which we attach to lengths or intervals of time depend upon the definitions and postulates which we adopt. The words "absolute" space or time intervals are devoid of meaning. As an illustration of what is meant Einstein discussed two possible ways of measuring the length of a rod when it is moving in the direction of its own length with a uniform velocity, that is, after having adopted a scale of length, two ways of assigning a number to the length of the rod concerned. One method is to imagine the observer moving with the rod, applying along its length the measuring scale, and reading off the positions of the ends of the rod. Another method would be to have two observers at rest on the body with reference to which the rod has the uniform velocity, so stationed along the line of motion of the rod that as the rod moves past them they can note simultaneously on a stationary measuring scale the positions of the two ends of the rod. Einstein showed that, accepting two postulates which need no defense at this time, the two methods of measurements would lead to different numerical values, and, further, that the divergence of the two results would increase as the velocity of the rod was increased. In assigning a number, therefore, to the length of a moving rod, one must make a choice of the method to be used in measuring it. Obviously the preferable method is to agree that the observer shall move with the rod, carrying his measuring instrument with him. This disposes of the problem of measuring space relations. The observed fact that, if we measure the length of the rod on different days, or when the rod is lying in different positions, we always obtain the same value offers no information concerning the "real" length of the rod. It may have changed, or it may not. It must always be remembered that measurement of the length of a rod is

simply a process of comparison between it and an arbitrary standard, *e.g.*, a meter-rod or yard-stick. In regard to the problem of assigning numbers to intervals of time, it must be borne in mind that, strictly speaking, we do not "measure" such intervals, *i.e.*, that we do not select a unit interval of time and find how many times it is contained in the interval in question. (Similarly, we do not "measure" the pitch of a sound or the temperature of a room.) Our practical instruments for assigning numbers to time-intervals depend in the main upon our agreeing to believe that a pendulum swings in a perfectly uniform manner, each vibration taking the same time as the next one. Of course we cannot *prove* that this is true, it is, strictly speaking, a definition of what we mean by equal intervals of time; and it is not a particularly good definition at that. Its limitations are sufficiently obvious. The best way to proceed is to consider the concept of uniform velocity, and then, using the idea of some entity having such a uniform velocity, to define equal intervals of time as such intervals as are required for the entity to traverse equal lengths. These last we have already defined. What is required in addition is to adopt some moving entity as giving our definition of uniform velocity. Considering our known universe it is self evident that we should choose as our definition of uniform velocity the velocity of light, since this selection could be made by an observer anywhere in our universe. Having agreed then to illustrate by the words "uniform velocity" that of light, our definition of equal intervals of time is complete. This implies, of course, that there is no uncertainty on our part as to the fact that the velocity of light always has the same value at any one point in the universe to any observer, quite regardless of the source of light. In other words, the postulate that this is true underlies our definition. Following this method Einstein developed a system of measuring both space and time intervals. As a matter of fact his system is identically that which we use in daily life with reference to events here on the earth. He further showed that if a man were to measure the length of a rod, for instance, on the earth and then were able to carry the rod and his measuring apparatus to Mars, the sun, or to Arcturus, he would obtain the same numerical value for the length in all places and at all times. This doesn't mean that any statement is implied as to whether the length of the rod has remained unchanged or not; such words do not have any meaning—remember that we cannot speak of true length. It is thus clear that an observer living on the earth would have a definite system of units in terms of which to express space and time intervals, *i.e.*, he would have a definite system of space coördinates (x, y, z) and a definite time coördinate (t); and similarly an observer living on Mars would have his system of coördinates (x', y', z', t'). Provided that one observer has a definite uniform velocity with reference to the other, it is a comparatively simple matter to deduce the mathematical relations between the two sets of coördinates. When Einstein did this, he arrived at the same transformation formulae as those used by Lorentz in his development of Maxwell's equations. The latter had shown that, using these formulae, the form of the laws for all electromagnetic phenomena maintained the same form; so Einstein's method proves that using

his system of measurement an observer, anywhere in the universe, would as the result of his own investigation of electromagnetic phenomena arrive at the same mathematical statement of them as any other observer, provided only that the relative velocity of the two observers was uniform.

Einstein discussed many other most important questions at this time; but it is not necessary to refer to them in connection with the present subject. So far as this is concerned, the next important step to note is that taken in the famous address of Minkowski, in 1908, on the subject of "Space and Time." It would be difficult to overstate the importance of the concepts advanced by Minkowski. They marked the beginning of a new period in the philosophy of physics. I shall not attempt to explain his ideas in detail, but shall confine myself to a few general statements. His point of view and his line of development of the theme are absolutely different from those of Lorentz or of Einstein; but in the end he makes use of the same transformation formulæ. His great contribution consists in giving us a new geometrical picture of their meaning. It is scarcely fair to call Minkowski's development a picture; for to us a picture can never have more than three dimensions, our senses limit us; while his picture calls for perception of four dimensions. It is this fact that renders any even semi-popular discussion of Minkowski's work so impossible. We can all see that for us to describe any event a knowledge of four coördinates is necessary, three for the space specification and one for the time. A complete picture could be given then by a point in four dimensions. All four coördinates are necessary: we never observe an event except at a certain time, and we never observe an instant of time except with reference to space. Discussing the laws of electromagnetic phenomena, Minkowski showed how in a space of four dimensions, by a suitable definition of axes, the mathematical transformation of Lorentz and Einstein could be described by a rotation of the set of axes. We are all accustomed to a rotation of our ordinary cartesian set of axes describing the position of a point. We ordinarily choose our axes at any location on the earth as follows: one vertical, one east and west, one north and south. So if we move from any one laboratory to another, we change our axes; they are always orthogonal, but in moving from place to place there is a rotation. Similarly Minkowski showed that if we choose four orthogonal axes at any point on the earth, according to his method, to represent a space-time point using the method of measuring space and time intervals as outlined by Einstein; and, if an observer on Arcturus used a similar set of axes and the method of measurement which he naturally would, the set of axes of the latter could be obtained from those of the observer on the earth by a pure rotation (and naturally a transfer of the origin). This is a beautiful geometrical result. To complete my statement of the method, I must add that instead of using as his fourth axis one along which numerical values of time are laid off, Minkowski defined his fourth coördinate as the product of time and the imaginary constant, the square root of minus one. This introduction of imaginary quantities might be expected, possibly, to introduce difficulties; but, in reality, it is the very essence of the simplicity of the geometrical description just given

of the rotation of the sets of axes. It thus appears that different observers situated at different points in the universe would each have their own set of axes, all different, yet all connected by the fact that any one can be rotated so as to coincide with any other. This means that there is no one direction in the four dimensional space that corresponds to time for all observers. Just as with reference to the earth there is no direction which can be called vertical for all observers living on the earth. In the sense of an *absolute* meaning the words "up and down," "before and after," "sooner or later," are entirely meaningless.

This concept of Minkowski's may be made clearer, perhaps, by the following process of thought. If we take a section through our three dimensional space, we have a plane, *i.e.*, a two dimensional space. Similarly, if a section is made through a four dimensional space, one of three dimensions is obtained. Thus, for an observer on the earth a definite section of Minkowski's four dimension space will give us our ordinary three dimensional one; so that this section will, as it were, break up Minkowski's space into our space and give us our ordinary time. Similarly, a different section would have to be used for the observer on Arcturus; but by a suitable selection he would get his own familiar three dimensional space and his own time. Thus the space defined by Minkowski is completely isotropic in reference to measured lengths and times, there is absolutely no difference between any two directions in an absolute sense; for any particular observer, of course, a particular section will cause the space to fall apart so as to suit his habits of measurement; any section, however, taken at random will do the same thing for some observer somewhere. From another point of view, that of Lorentz and Einstein, it is obvious that, since this four dimensional space is isotropic, the expression of the laws of electromagnetism take identical mathematical forms when expressed by any observer.

The question of course must be raised as to what can be said in regard to phenomena which so far as we know do not have an electromagnetic origin. In particular what can be done with respect to gravitational phenomena? Before, however, showing how this problem was attacked by Einstein—and the fact that the subject of my address is Einstein's work on gravitation shows that ultimately I shall explain this—I must emphasize another feature of Minkowski's geometry. To describe the space-time characteristics of any event a point, defined by its four coördinates, is sufficient; so, if one observes the life-history of any entity, *e.g.*, a particle of matter, a light-wave, etc., he observes a sequence of points in the space-time continuum; that is, the life-history of any entity is described fully by a line in this space. Such a line was called by Minkowski a "world-line." Further, from a different point of view, all of our observations of nature are in reality observations of coincidences, *e.g.*, if one reads a thermometer, what he does is to note the coincidence of the end of the column of mercury with a certain scale division on the thermometer tube. In other words, thinking of the world-line of the end of the mercury column and the world-line of the scale division, what we have observed was the intersection or crossing of these lines. In a similar manner any observation may be ana-

lyzed; and remembering that light rays, a point on the retina of the eye, etc., all have their world-lines, it will be recognized that it is a perfectly accurate statement to say that every observation is the perception of the intersection of world-lines. Further, since all we know of a world-line is the result of observations, it is evident that we do not know a world-line as a continuous series of points, but simply as a series of discontinuous points, each point being where the particular world-line in question is crossed by another world-line.

It is clear, moreover, that for the description of a world-line we are not limited to the particular set of four orthogonal axes adopted by Minkowski. We can choose any set of four dimensional axes we wish. It is further evident that the mathematical expression for the coincidence of two points is absolutely independent of our selection of reference axes. If we change our axes, we will change the coördinates of both points simultaneously, so that the question of axes ceases to be of interest. But our so called laws of nature are nothing but descriptions in mathematical language of our observations; we observe only coincidences; a sequence of coincidences when put in mathematical terms takes a form which is independent of the selection of reference axes; therefore the mathematical expression of our laws of nature, of every character, must be such that their form does not change if we make a transformation of axes. This is a simple but far-reaching deduction.

There is a geometrical method of picturing the effect of a change of axes of reference, *i.e.*, of a mathematical transformation. To a man in a railway coach the path of a drop of water does not appear vertical, *i.e.*, it is not parallel to the edge of the window; still less so does it appear vertical to a man performing manœuvres in an airplane. This means that whereas with reference to axes fixed to the earth the path of the drop is vertical; with reference to other axes, the path is not. Or, stating the conclusion in general language, changing the axes of reference (or effecting a mathematical transformation) in general changes the shape of any line. If one imagines the line forming a part of the space, it is evident that if the space is deformed by compression or expansion the shape of the line is changed, and if sufficient care is taken it is clearly possible, by deforming the space, to make the line take any shape desired, or, better stated, any shape specified by the previous change of axes. It is thus possible to picture a mathematical transformation as a deformation of space. Thus I can draw a line on a sheet of paper or of rubber and by bending and stretching the sheet, I can make the line assume a great variety of shapes; each of these new shapes is a picture of a suitable transformation.

Now, consider world-lines in our four dimensional space. The complete record of all our knowledge is a series of sequences of intersections of such lines. By analogy I can draw in ordinary space a great number of intersecting lines on a sheet of rubber; I can then bend and deform the sheet to please myself; by so doing I do not introduce any new intersections nor do I alter in the least the sequence of intersections. So in the space of our world-lines, the space may be deformed in any imaginable manner without introducing any new intersections or changing the sequence of the existing intersections. It is this

sequence which gives us the mathematical expression of our so-called experimental laws; a deformation of our space is equivalent mathematically to a transformation of axes, consequently we see why it is that the form of our laws must be the same when referred to any and all sets of axes, that is, must remain unaltered by any mathematical transformation.

Now, at last, we come to gravitation. We cannot imagine any world-line simpler than that of a particle of matter left to itself; we shall therefore call it a "straight" line. Our experience is that two particles of matter attract one another. Expressed in terms of world-lines, this means that, if the world-lines of two isolated particles come near each other, the lines, instead of being straight, will be deflected or bent in towards each other. The world-line of any one particle is therefore deformed; and we have just seen that a deformation is the equivalent of a mathematical transformation. In other words, for any one particle it is possible to replace the effect of a gravitational field at any instant by a mathematical transformation of axes. The statement that this is always possible for any particle at any instant is Einstein's famous "Principle of Equivalence."

Let us rest for a moment, while I call attention to a most interesting coincidence, not to be thought of as an intersection of world-lines. It is said that Newton's thoughts were directed to the observation of gravitational phenomena by an apple falling on his head; from this striking event he passed by natural steps to a consideration of the universality of gravitation. Einstein in describing his mental process in the evolution of his law of gravitation says that his attention was called to a new point of view by discussing his experiences with a man whose fall from a high building he had just witnessed. The man fortunately suffered no serious injuries and assured Einstein that in the course of his fall he had not been conscious in the least of any pull downward on his body. In mathematical language, with reference to axes moving with the man the force of gravity had disappeared. This is a case where by the transfer of the axes from the earth itself to the man, the force of the gravitational field is annulled. The converse change of axes from the falling man to a point on the earth could be considered as introducing the force of gravity into the equations of motion. Another illustration of the introduction into our equations of a force by means of a change of axes is furnished by the ordinary treatment of a body in uniform rotation about an axis. For instance, in the case of a so-called conical pendulum, that is, the motion of a bob suspended from a fixed point by a string, which is so set in motion that the bob describes a horizontal circle and the string therefore describes a circular cone, if we transfer our axes from the earth and have them rotate around the vertical line through the fixed point with the same angular velocity as the bob, it is necessary to introduce into our equations of motion a fictitious "force" called the centrifugal force. No one ever thinks of this force other than as a mathematical quantity introduced into the equations for the sake of simplicity of treatment; no physical meaning is attached to it. Why should there be to any other so-called "force," which, like centrifugal force, is independent of the nature of the matter?

Again, here on the earth our sensation of weight is interpreted mathematically by combining expressions for centrifugal force and gravity; we have no distinct sensation for either separately. Why then is there any difference in the essence of the two? why not consider them both as brought into our equations by the agency of mathematical transformations? This is Einstein's point of view.

Granting, then, the principle of equivalence, we can so choose axes at any point at any instant that the gravitational field will disappear; these axes are therefore of what Eddington calls the "Galilean" type, the simplest possible. Consider, that is, an observer in a box, or compartment, which is falling with the acceleration of the gravitational field at that point. He would not be conscious of the field. If there were a projectile fired off in this compartment, the observer would describe its path as being straight. In this space the infinitesimal interval between two space-time points would then be given by the formula

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$$

where ds is the interval and x_1, x_2, x_3, x_4, are coördinates. If we make a mathematical transformation, *i.e.*, use another set of axes, this interval would obviously take the form

$$ds^2 = g_{11}\,dx_1^2 + g_{22}\,dx_2^2 + g_{33}\,dx_3^2 + g_{44}\,dx_4^2 + 2g_{12}\,dx_1\,dx_2 + \text{, etc.}$$

where x_1, x_2, x_3, and x_4 are now coördinates referring to the new axes. This relation involves ten coefficients, the coefficients defining the transformation.

But of course a certain dynamical value is also attached to the g's, because by the transfer of our axes from the Galilean type we have made a change which is equivalent to the introduction of a gravitational field; and the g's must specify the field. That is, these g's are the expression of our experiences, and hence their values cannot depend upon the use of any special axes; the values must be the same for all selections. In other words, whatever function of the coördinates any one g is for one set of axes, if other axes are chosen, this g must still be the same function of the new coördinates. There are ten g's defined by differential equations; so we have ten covariant equations. Einstein showed how these g's could be regarded as generalized potentials of the field. Our own experiments and observations upon gravitation have given us a certain knowledge concerning its potential; that is, we know a value for it which must be so near the truth that we can properly call it at least a first approximation. Or, stated differently, if Einstein succeeds in deducing the rigid value for the gravitational potential in any field, it must degenerate to the Newtonian value for the great majority of cases with which we have actual experience. Einstein's method, then, was to investigate the functions (or equations) which would satisfy the mathematical conditions just described.

A transformation from the axes used by the observer in the falling box may be made so as to introduce into the equations the gravitational field recognized by an observer on the earth near the box; but this, obviously, would not be the general gravitational field, because the field changes as one moves over the surface of the earth. A solution found, therefore, as just indicated would not

be the one sought for the general field; and another must be found which is less stringent than the former but reduces to it as a special case. He found himself at liberty to make a selection from among several possibilities, and for several reasons chose the simplest solution. He then tested this decision by seeing if his formulæ would degenerate to Newton's law for the limiting case of velocities small when compared with that of light, because this condition is satisfied in those cases to which Newton's law applies. His formulæ satisfied this test, and he therefore was able to announce a "law of gravitation," of which Newton's was a special form for a simple case.

To the ordinary scholar the difficulties surmounted by Einstein in his investigations appear stupendous. It is not improbable that the statement which he is alleged to have been made to his editor, that only ten men in the world could understand his treatment of the subject, is true. I am fully prepared to believe it, and wish to add that I certainly am not one of the ten. But I can also say that, after a careful and serious study of his papers, I feel confident that there is nothing in them which I cannot understand, given the time to become familiar with the special mathematical processes used. The more I work over Einstein's papers, the more impressed I am, not simply by his genius in viewing the problem, but also by his great technical skill.

Following the path outlined, Einstein, as just said, arrived at certain mathematical laws for a gravitational field, laws which reduced to Newton's form in most cases where observations are possible, but which led to different conclusions in a few cases, knowledge concerning which we might obtain by careful observations. I shall mention a few deductions from Einstein's formulæ.

1. If a heavy particle is put at the center of a circle, and, if the length of the circumference and the length of the diameter are measured, it will be found that their ratio is not π (3.14159). In other words the geometrical properties of space in such a gravitational field are not those discussed by Euclid; the space is, then, non-Euclidean. There is no way by which this deduction can be verified, the difference between the predicted ratio and π is too minute for us to hope to make our measurements with sufficient exactness to determine the difference.

2. All the lines in the solar spectrum should with reference to lines obtained from terrestrial sources be displaced slightly towards longer wave-lengths. The amount of displacement predicted for lines in the blue end of the spectrum is about one hundredth of an Ångstrom unit, a quantity well within experimental limits. Unfortunately, as far as the testing of this prediction is concerned, there are several physical causes which are also operating to cause displacement of the spectrum-lines; and so at present a decision cannot be rendered as to the verification. St. John and other workers at the Mount Wilson Observatory have the question under investigation.

3. According to Newton's law an isolated planet in its motion around a central sun would describe, period after period, the same elliptical orbit; whereas Einstein's laws lead to the prediction that the successive orbits traversed would not be identically the same. Each revolution would start the

planet off on an orbit very approximately elliptical, but with the major axis of the ellipse rotated slightly in the plane of the orbit. When calculations were made for the various planets in our solar system, it was found that the only one which was of interest from the standpoint of verification of Einstein's formulæ was Mercury. It has been known for a long time that there was actually such a change as just described in the orbit of Mercury, amounting to 574″ of arc per century; and it has been shown that of this a rotation of 532″ was due to the direct action of other planets, thus leaving an unexplained rotation of 42″ per century. Einstein's formulæ predicted a rotation of 43″, a striking agreement.

4. In accordance with Einstein's formulæ a ray of light passing close to a heavy piece of matter, the sun, for instance, should experience a sensible deflection in towards the sun. This might be expected from "general" considerations. A light ray, is of course, an illustration of energy in motion; energy and mass are generally considered to be identical in the sense that an amount of energy E has the mass E/c^2 where c is the velocity of light; and consequently a ray of light might fall within the province of gravitation and the amount of deflection to be expected could be calculated by the ordinary formula for gravitation. Another point of view is to consider again the observer inside the compartment falling with the acceleration of the gravitational field. To him the path of a projectile and a ray of light would both appear straight; so that, if the projectile had a velocity equal to that of light, it and the light wave would travel side by side. To an observer outside the compartment, *e.g.*, to one on the earth, both would then appear to have the same deflection owing to the sun. But how much would the path of the projectile be bent? what would be the shape of its parabola? One might apply Newton's law; but, according to Einstein's formulæ, Newton's law should be used only for small velocities. In the case of a ray passing close to the Sun it was deduced that according to Einstein's formula there should be a deflection of 1″.75 whereas Newton's law of gravitation predicted half this amount. Careful plans were made by various astronomers, to investigate this question at the solar eclipse last May, and the result announced by Dyson, Eddington and Crommelin, the leaders of astronomy in England, was that there was a deflection of 1″.9. Of course the detection of such a minute deflection was an extraordinarily difficult matter, so many corrections had to be applied to the original observations; but the names of the men who record the conclusions are such as to inspire confidence. Certainly any effect of refraction seems to be excluded.

It is thus seen that the formulæ deduced by Einstein have been confirmed in a variety of ways and in a most brilliant manner. In connection with these formulæ one question must arise in the minds of everyone: by what process, where in the course of the mathematical development, does the idea of mass reveal itself? It was not in the equations at the beginning and yet here it is at the end. How does it appear? As a matter of fact it is first seen as a constant of integration in the discussion of the problem of the gravitational field due to a single particle; and the identity of this constant with mass is proved when

one compares Einstein's formulæ with Newton's law which is simply its degenerated form. This mass, though, is the mass of which we become aware through our experiences with weight; and Einstein proceeded to prove that this quantity which entered as a constant of integration in his ideally simple problem also obeyed the laws of conservation of mass and conservation of momentum when he investigated the problems of two and more particles. Therefore Einstein deduced from his study of gravitational fields the well-known properties of matter which form the basis of theoretical mechanics. A further logical consequence of Einstein's development is to show that energy has mass, a concept with which every one nowadays is familiar.

The description of Einstein's method which I have given so far is simply the story of one success after another; and it is certainly fair to ask if we have at last reached finality in our investigation of nature, if we have attained to truth. Are there no outstanding difficulties? Is there no possibility of error? Certainly not until all the predictions made from Einstein's formulæ have been investigated can much be said; and further, it must be seen whether any other lines of argument will lead to the same conclusions. But without waiting for all this there is at least one difficulty which is apparent at this time. We have discussed the laws of nature as independent, in their form, of reference axes, a concept which appeals strongly to our philosophy; yet it is not at all clear, at first sight, that we can be justified in our belief. We cannot imagine any way by which we can become conscious of the translation of the earth in space; but by means of gyroscopes we can learn a great deal about its rotation on its axis. We could locate the positions of its two poles, and by watching a Foucault pendulum or a gyroscope we can obtain a number which we interpret as the angular velocity of rotation of axes fixed in the earth: angular velocity with reference to what? Where is the fundamental set of axes? This is a real difficulty. It can be surmounted in several ways. Einstein himself has outlined a method which in the end amounts to assuming the existence on the confines of space of vast quantities of matter, a proposition which is not attractive. de Sitter has suggested a peculiar quality of the space to which we refer our space-time coördinates. The consequences of this are most interesting, but no decision can as yet be made as to the justification of the hypothesis. In any case we can say that the difficulty raised is not one that destroys the real value of Einstein's work.

In conclusion I wish to emphasize the fact, which should be obvious, that Einstein had not attempted any explanation of gravitation; he has been occupied with the deduction of its laws. These laws, together with those of electromagnetic phenomena, comprise our store of knowledge. There is not the slightest indication of a mechanism, meaning by that a picture in terms of our senses. In fact what we have learned has been to realize that our desire to use such mechanisms is futile.

VARIATION OF TRANSPARENCY TO TOTAL RADIATION WITH TEMPERATURE OF SOURCE.[1]

BY S. LEROY BROWN.

SINCE the character of the radiation from a black body is so changed by a rise in temperature that it becomes richer in the shorter wave-lengths, a higher transparency is to be expected for substances that have selective transmission in the visible spectrum, when the temperature of the radiating source is increased. Substances that do not show a marked selective transmission for the shorter wave-lengths would hardly be expected to show as much increase of transparency with rise of temperature of the radiating source. Glass, for example, transmits only a small amount of the total radiation from a black body at a moderate temperature, but transmits a large fractional part of the radiation from the sun. Rock salt, on the other hand, would not be expected to show such marked increase of transparency since it transmits the longer wave-lengths.

It is the purpose of this paper to give the results of experiments in measuring the total radiation transmitted by several substances, and to show how the transparency increases with the temperature of the radiating source. These results show a decided increase of transparency with temperature for all substances tried. They also show that some substances may be very transparent within the visible spectrum while transmitting only a very small fractional part of the total black body radiation.

The plan of these experiments was to compare the intensity of the incident radiation with the intensity of the transmitted radiation by allowing them to be focused from the metal surface of a concave mirror on a sensitive resistance element; the intensity being measured in terms of the decrease in the resistance of the element as the total radiation focused upon it is increased. The source of the radiation for temperatures up to 1200° C. was a tubular electric furnace.

Temperature of Radiating Source.	Per Cent. of Total Radiation Transmitted by				
	Glass (1.2 mm.).	Rock Salt (11 mm.).	Mica (1 mm.).	Cellul^{0}id (2 mm.).	Paraffin (.3 mm.).
200° C.	5.8	35			
300°	3.1	36			
400°	10.8	37	7		
500°	14	38	10		
600°	18	39	13		
700°	22	40.1	16		
800°	27	41.3	19	3.8	7
900°	32	43	23	6.2	8
1000°	40	45	28	9	9
1100°	49	47.3	34.5	12.8	10.7
1200°	61	50.5	40	16	12

UNIVERSITY OF TEXAS.

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

THE DISSIPATION OF HEAT BY VARIOUS SURFACES IN STILL AIR.[1]

BY T. S. TAYLOR.

THE investigation herein described was undertaken for the purpose of obtaining some definite data on the amounts of heat that are transmitted through sheet tin, such as is used in hot air pipes, and dissipated to the surrounding air under various surface conditions. Three vessels each 10 cm. in diameter and 50 cm. long were prepared from sheet tin, one being left bare, one having 0.33 mm. sheet asbestos put tightly over its surface, and one covered loosely with sheet asbestos.

The vessels were filled with hot water and allowed to cool when placed horizontally in similar positions with respect to surroundings. Disturbances due to air currents were avoided as much as possible. Observations were taken of the surrounding air temperature, the temperature of the vessels as measured by a thermometer inserted through the middle of the side and having its bulb at the axis of the vessel, and the corresponding times. From these results cooling curves were obtained for each vessel which were found to satisfy fairly closely the ordinary law of cooling

$$\theta = \theta_0 e^{-at}$$

The values for a, the measure of the rate of cooling, were found to be .0046 min.$^{-1}$, and 0.0066 min.$^{-1}$, and 0.0073 min.$^{-1}$ for the bare surface, loosely covered surface, and tightly covered surface, respectively. It is thus seen that the loss of heat by a bare tin surface is only 70 per cent. as much as for the same surface when covered loosely with sheet asbestos, and only 63 per cent. as much as for the same material when covered tightly with sheet asbestos. The cooling constant for the bare tin when coated with aluminum paint was found to be 0.0050 min.$^{-1}$ which is slightly greater than for the bare surface. By dusting the surfaces with a layer of coal ashes, the following values of a were found; for the bare surface 0.0048 min.$^{-1}$, and for the surface covered tightly with asbestos 0.0069 min.$^{-1}$. It is seen that the dust has little influence on the bare surface but considerable on the others.

These results show that it is far more economical and efficient to install hot air furnace pipes and similar apparatus in the uncovered state. Not only the expense of labor and material connected with initial cost are saved but much less heat is lost afterwards.

THE MEASUREMENT OF THE THERMAL EXPANSION OF METALS AT ORDINARY TEMPERATURES.[1]

BY CHARLES D. HODGMAN.

A METHOD of measuring thermal expansion has been developed which is especially adapted for use in the case of short samples of metal and at temperatures ranging from 0° to 100° C. The method was found satisfactory

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

and has since been used in the case of longer samples in preference to any other arrangement for work in the same temperature range.

A line comparator provided with two micrometer microscopes is employed. The comparator is furnished with a rolling carriage carrying two platforms with adjustments for bringing a bar or scale into focus under the microscopes. A suitably insulated metal box containing the sample is clamped directly to one of the platforms. The sample is drilled and tapped near the ends to receive special short index pegs which project vertically upward. The box is provided with an easily removable cover with large openings for the index posts. A sheet of very thin rubber closes the openings around these posts making it possible to circulate water or steam in the box. The second platform may carry a standard scale with arrangements for maintaining at constant temperature; a reasonable degree of accuracy may be obtained, however, by making corrections for the variations in temperature of the base of the comparator when these changes are small.

The procedure described above has been followed in determining the mean coefficient of linear expansion between 10 and 100° C. of a large number of samples of alloys. A series of measurements were also made on cast and wrought magnesium for the same temperature range. The average of ten independent determinations in each case gave the following results:

Average coefficient of linear expansion	20–100° C.
Cast magnesium	0.00002686
Wrought magnesium	0.00002673

The average deviation of the mean for these measurements, 0.12 per cent. and 0.15 per cent. respectively, indicates the accuracy obtained.

CASE SCHOOL OF APPLIED SCIENCE.

A METHOD FOR DETERMINING THE PHOTOGRAPHIC ABSORPTION OF LENSES.[1]

BY G. W. MOFFITT.[2]

THE axial photographic transmission coefficient of a lens system for a given set of conditions will here be defined as the ratio of the light flux of photographic quality in the image of a small object on the axis of the system to the light flux of photographic quality that would reach the image were there no losses of any kind in transmission through the system other conditions remaining the same.

The method employed in determing this coefficient involves the photographing of the lens image of a controlled circular source of approximate daylight quality and radiating according to the Lambert cosine law. Immediately after the completion of the lens exposure strips of the plate on both sides and adjacent to the lens image are impressed in a series of steps by direct action of the same controlled source in such a manner that the exposure in each step is

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

[2] Communication No. 87 from the Research Laboratory of the Eastman Kodak Company

continuous from beginning to end. From the developed plate the ratio of the times of exposure for equal densities in strips and image may be found. This ratio together with the dimensions of the apparatus and the lens gives sufficient data for the determination of the transmission coefficient of the lens for light of photographic quality. If dependable results are to be obtained, care must be exercised in screening the plate from the action of stray light, from nearby objects during the sensitometer exposure and from barrel reflections during the lens exposure. These later may easily be eliminated by using a stop considerably less than the maximum thereby eliminating the reflected light from cell rings and lens edges.

The entire light flux directed towards the entrance pupil of a lens system from a small element of source dS_0 on and perpendicular to the axis of the system and radiating according to the Lambert cosine law is

$$L = \pi I_0 dS_0 \sin^2 U,$$

in which I_0 is a constant characteristic of the source and U is the angle subtended at the source by the radius of the entrance pupil. Were there no losses this flux would ultimately be uniformly distributed in the image whose size is controlled by the magnification. The flux density in the image (angle of incidence on plate disregarded) would therefore be

$$I_i = \frac{\pi I_0 \sin^2 U}{Y^2},$$

in which Y is the lateral magnification assumed positive for real images.

Now if a photographic plate be directly exposed to the action of radiation from a plane circular source parallel to the plate and radiating according to the Lambert cosine law the flux density at the plate will be

$$I_s = \frac{\pi I_0 R^2}{R^2 + l^2},$$

in which I_0 is the constant characteristic and R the radius of the source, and l is the perpendicular distance from the source to the plate.

From these fundamental equations the expression for the transmission coefficient may easily be found to be

$$T = \frac{t_s}{t_i} \cdot \frac{R^2 Y^2}{(R^2 + 1^2)\, s\,\text{in}^2 U} = \frac{t_s}{t_1} \cdot \frac{R^2}{R^2 + l^2} \left[Y^2 + \frac{[Y(f+d)+f]^2}{h^2} \right],$$

in which t_s/t_i is the ratio of exposure times for equal densities in strips and image, d is the distance from the first principal point of the lens system to the entrance pupil considered positive when the entrance pupil is farther away from the source than is the first principal point, and h is the radius of the pupil. The equivalent focal length of the system is represented by f and the other symbols have the meanings already stated. When the object distance is large Y is small and may become negligible in comparison with f in which case the equation reduces to the following simple form

$$T = \frac{t_s}{t_i} \cdot \frac{4R^2 S^2}{R^2 + l^2},$$

in which S is the F-number of the lens.

Because of inherent irregularities in the photographic plate quite a number of exposures must be made in order to arrive at a good average value of the transmission coefficient. The preliminary results obtained have been found to be in good general agreement with those found by visual methods[1] as well as with values deduced from the theory of reflection and absorption.

EASTMAN KODAK COMPANY,
ROCHESTER, N. Y.
December 6, 1919.

DEFECTS IN CENTERED QUADRIC LENSES.[2]

BY IRWIN ROMAN.

THE purpose of this paper is to consider the theory of aberrations in a system in which the lenses are centered quadrics of revolution. The method used is Abbe's method of optical invariants, and all terms of higher than the second degree in the small quantities are neglected, as for the usual treatment for the spherical case. The curvature of the field, the relative error in zonal magnification and the angular coma as seen from the diaphragm are found to be independent of the eccentricity of the generating conic, to the selected degree of approximation. The longitudinal aberration depends on the eccentricities and is found to be given by

$$L = \tfrac{1}{2}\Sigma \frac{\epsilon^2 h^2}{r_0} - \frac{1}{2\theta_n^2}\Sigma Q_0^2 h^3 \left(\frac{\epsilon^2 h}{r_0} \Delta \frac{1}{\mu} + \Delta \frac{\theta}{\mu} \right),$$

where ϵ = eccentricity of the generating conic.
h = semi-aperture height.
r_0 = axial curvature of the lens surface.
θ_n = angular aperture, measured in final medium.
θ = angular aperture, measured in medium concerned.
μ = index of refraction.
Q_0 = the Abbe invariant = $(k - \kappa)$.
Δ = "increment in" for passage through a refracting surface.

THE SINKER METHOD APPLIED TO THE RAPID AND ACCURATE DETERMINATION OF SPECIFIC GRAVITIES.[2]

BY N. W. CUMMINGS.

THE method is particularly applicable to the determinations of specific gravity, where (1) vast numbers of samples are to be run, (2) the error of a single determination must be less than 0.005 of one per cent., and (3) the variation of specific gravity among the different samples is not greater than 2 or 3 in the second decimal place. It was developed especially for work with

[1] P. G. Nutting, Astrophys. Jour., 40, pp. 33–42, 1914. R. W. Cheshire, Proc. Opt. Con., pp. 34–40, 1912.

[2] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

sea-water. The apparatus required consists of a high-grade balance, a sinker, preferably of about 25 c.c. volume, a fine wire suspension for the sinker, a containing vessel, a special form of which has been designed, and a syphon arrangement for conveniently introducing and removing the sample. It is shown that the $S_{t/04}$ of sea-water can be computed with a error of less than 0.005 of one per cent. by the formula

$$S_{t/04} = \rho_d + \frac{W_d - W}{V} + S^{t/04}\,\alpha(20 - t)$$

where $S_{t/04}$ is the density of sea-water at $t°$, ρ_d the density of distilled water at 20°, W_d the weight needed to balance the sinker in distilled water at 20°, W the weight needed to balance the sinker in sea-water at $t°$, V the average volume of the sinker, which may be regarded as a constant and should be known with an accuracy of about one part in 2,500, and α is the thermal coefficient of cubical expansion of the material of the sinker. A method of constructing tables to expedite the work of correcting each specific gravity to the value corresponding to 0° is described.

SCRIPPS INSTITUTION FOR BIOLOGICAL RESEARCH OF THE UNIVERSITY OF CALIFORNIA.
LA JOLLA, CALIFORNIA,
December 1, 1919.

AMPLIFICATION OF CURRENTS IN THE BUNSEN FLAME.[1]

BY C. W. HEAPS.

WHEN a current of electricity flows between two clean platinum electrodes in a Bunsen flame it is found that the potential gradient in the flame is usually much greater near the negative electrode than at any other place. This fall of potential near the cathode has the effect of making the resistance of the flame very great. Now if salt or lime is put on the cathode the large cathode fall of potential disappears, and the effective resistance of the flame is made perhaps a hundred times smaller. Apparently, by increasing the electron emission from the heated cathode the negative drop may be diminished. It might be expected, therefore, that by diminishing the electron emission of the lime or salt on the hot cathode the negative drop could be made to appear once more and the flame resistance consequently much increased again.

In the case of vacuum tubes of the audion type the electron emission of the hot filament is much affected by the potential of the grid. Experiments have been conducted to see whether a similar effect could be obtained in a flame by the introduction of a third terminal, or grid, near the lime-coated cathode, this third terminal being given various potentials with respect to the cathode. The characteristic curves obtained giving the current through the flame as a function of the grid potential are very similar in form to those obtained with the three electrode vacuum amplifiers. The most satisfactory arrangement of

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

electrodes in the flame consisted of (1) a clean horizontal platinum wire near the base of the flame to serve as the positive terminal, or plate, (2) a horizontal wire with about a centimeter of the end bent upwards to serve as cathode, and (3) a coil of three or four turns of wire serving as a grid and placed over the upturned end of the cathode. On the tip of the cathode a bit of sealing wax was burned, leaving a residue which when heated acted as an efficient source of electrons.

The direct current characteristics of this device have been studied in the way usual with audions. With a battery of 130 volts in the plate circuit and variations of grid potential of about 15 volts the change of plate current is several microamperes, while the input current is about 10^{-8} amperes. The energy amplification, calculated by comparing the watt loss in an external resistance of the plate circuit with the energy input of the grid circuit, could be made in some cases as large as 450 or 500. The voltage amplification was usually small.

For alternating grid potentials this device does not function in such a simple manner. A flame behaves for high frequency currents like a capacity rather than like a pure resistance. Phenomena also appear which are analogous to polarization effects when alternating grid potentials are used.

RICE INSTITUTE,
HOUSTON, TEXAS,
November 19, 1919.

A NEW TYPE OF NON-INDUCTIVE RESISTANCE.[1]

BY H. L. DODGE.

A NEW type of non-inductive winding has been devised which lends itself to the commercial production of resistances in the form of current carrying rheostats for general laboratory use as well as precision standards of high ohmic resistance. The conductor is supported on two parallel rods or cores which may be at any convenient distance from each other. It is in this respect that the new winding is superior to the so-called Curtis coils which must be wound upon cores separated by only a narrow slit. The winding follows the formula:

$$\cdots\cdots o\,x\,o\,x\,x\,o\,x\,o\,o\,x\,o\,x\,x\,o\,x\,o\,o\,x\,o\,x\,x\,o\,x\,o\cdots\cdots$$

in which "o" indicates that the conductor is carried from one support to the other without crossing through the space between the supports while "x" indicates that the conductor is carried across the space between the supports when passed from one to the other. Any group of eight operations produces one "unit" of the winding.

A rough sketch, made according to the formula, is useful in showing the characteristics of the winding. It will be found that every part of the conductor is closely adjacent to a similarly placed part carrying the current in the opposite

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

direction. Thus a very small coefficient of self induction is secured. The capacity is also small as there is but one layer of wire and the potential difference between adjacent strands is small. For the same reason it is possible to use oxide-covered wire for rheostats.

SOME LABORATORY USES FOR THE CONTACT RECTIFIER.[1]

BY J. C. JENSEN.

THE rectifying action of certain crystals has been studied by Pierce, Austin, Hartsough, Whittemore, Miss McDowell and others. Experiments have shown that for the type of telephones ordinarily used in radiotelegraphy a current of 5 to 10 microamperes is required for audibility. Galvanometers with a higher micro-volt sensibility than the best telephones are available in any laboratory and if an efficient rectifier can be found it should be possible to use a common galvanometer in series with it in place of the telephones in such experiments as the measurement of electrolytic conductivity and the coefficient of self-induction. The carbon-silicon rectifier described by Hartsough[2] was found most suitable for the purpose, four pairs being connected in the same manner as the cells of the Nodon valve. Contrary to the reports of some other observers, the multiple contacts secured by placing a piece of polished silicon against polished carbon gave greater rectification and less resistance than with a point contact, when adjusted for a pressure of about 5 grams per cm.^{2}.

Measurements of electrolytic conductivity with a high-frequency generators and telephones as compared with a 60-cycle current and galvanometer-rectifier showed the former to be the more reliable, the rectifier giving some variations

TABLE I.

$L = C[r(R_2 + R_4) + R_2R_3]$.

R_2.	R_3.	R_4.	r.	L (C=1 mfd.).
300	50	23.5	268.7	.1019 henries
500	100	23.5	99.	.1018
500	150	23.5	52.	.1021
400	50	23.5	194.	.1020
300	100	23.5	223.	.1021
300	150	23.5	177.	.1022
500	75	23.5	123.7	.1022
1000	60	23.5	41.	.1020
800	70	23.5	56.	.1021
700	100	23.5	44.5	.1021
700	80	23.5	63.5	.1019
600	60	23.5	106.	.1020
Mean				.1020

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

[2] PHYS. REV., Vol. IV., 1914, p. 306.

of 2 per cent. This was due in part to the polarizing effect of the low-frequency source. Tests with the rectifier and a more powerful high-frequency generator should give more constant values.

The observations in Table I. made by Mr. R. W. Hudson in determining the coefficient of self-induction of a coil by the Anderson-Maxwell method, the second adjustment being made by use of a rectifier and galvanometer, will show the regularity of results possible in every-day student work.

The following are the means of a set of four readings on each point of the dial in calibrating a variometer such as is used in radiotelegraphy, by means of the Maxwell method, using a Leeds & N. variable standard of self-inductance. A L. & N. vibration galvanometer, and a D'Arsonval galvanometer of 80 megohms sensitivity in series with the rectifier, were used interchangeably. Some of the variation is undoubtedly due to the difficulty in getting an exact reading of the standard self-inductance.

TABLE II.

Position.	L_2 Galvanometer.	L_2 Rectifier.
1	22.27	22.07 milli-henries
2	21.24	21.17
3	19.60	19.75
4	17.42	17.82
5	14.76	14.83
6	12.65	12.72
7	10.28	10.49
8	7.96	8.07

Table III. gives a comparison of results obtained with the same apparatus when using Maxwell's method of comparing mutual inductances. Results given are the average of a set of four readings in each case.

TABLE III.

Position.	M_2 Galvanometer.	M_2 Rectifier.
1	4.64	4.645 milli-henries
2	4.03	4.015
3	3.235	3.175
4	2.115	2.10
5	.965	.991

The values given lie within the accuracy demanded for most purposes and show the practicability of the method.

NEBRASKA WESLEYAN UNIVERSITY,
UNIVERSITY PLACE, NEBRASKA,
December 12, 1919.

DIFFICULTIES IN THE THEORY OF RAIN FORMATION.[1]

BY W. J. HUMPHREYS.

THE formation of rain drops has never been satisfactorily explained. The general assumptions seem to be (1) that as soon as saturation is passed, condensation occurs on the nuclei present; (2) that the larger droplets, owing to their lesser vapor tension, grow at the expense of the smaller; and (3) that the larger and faster falling droplets unite with enough others on their downward path to form full sized rain drops.

All this sounds very plausible, but it will not stand analysis. It is true that as the temperature falls below the dew point, condensation does occur on the dust particles and any other nuclei that may be present. But this at once introduces a formidable difficulty, that is, the number of such particles is so great that even all the water present could not develop them to a "falling" size. It is also true that the larger drops do grow at the expense of the smaller but, according to theory, at a rate far too slow to be effective in the process of rain production. Finally, even if a droplet should fall quite through a cloud layer, and actually coalesce with all particles in its path, the chance of its thus becoming a full-sized rain drop would be very small.

The chief steps in rain formation seem to be, (*a*) the continuous ascent of humid air; (*b*) the formation of cloud droplets on the nuclei of this air as soon as it cools below the dew point; (*c*) the filtering by these droplets of the next rising air; (*d*) the progressive condensation, in the midst of the rain cloud, on the relatively few droplets present in this automatically filtered air, and their consequent growth to "falling" size; (*e*) coalescence with other droplets.

A PHYSICAL THEORY OF OCEAN OR RESERVOIR TEMPERATURE DISTRIBUTIONS REGARDED AS EFFECTS OF SOLAR RADIATION, EVAPORATION AND THE RESULTING CONVECTION.[1]

BY GEO. F. MCEWEN.

ASSUME radiant energy to be absorbed in accordance with the well-known exponential function of the thickness of the medium traversed, and that a similar relation having a larger exponent holds for the removal of heat by evaporation, since the direct effect of the latter is confined to a comparitively thin surface layer, in spite of the mechanical disturbance usually present near the surface. It then follows that evaporation removes heat at a greater rate near the surface than can be directly supplied by radiation. This surface layer thus becomes colder than that underneath, and consequently tends to change places with it. This interchange may not be complete. That is, a fraction r of the cold upper layer may remain to mix with the fraction $(1 - r)$ of the rising warm layer. Similarly, the cold water replacing this warm layer tends to change places with the one underneath, and so on downward. Thus a convective circulation is generated consisting of the descent of relatively cold

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

water elements, and the ascent of relatively warm ones in which the difference in temperature decreases as the depth increases.

Two differential equations, one giving the rate of change of the temperature of the descending cold water, the other giving the temperature rate for the ascending warm water can be derived from these assumptions. Regarding the measured temperature as the mean of that of the intermixed warm and cold elements, a combination of the two differential equations into a single one can be obtained whose solution, subject to suitable boundary conditions gives the relation in such form as to admit of observational tests. The satisfactory qualitative agreement of one such solution with generally accepted facts led to preliminary estimates of the physical constants. The results thus found appear to justify an extended investigation of the theory.

Electromagnetic Induction and Relative Motion.[1]

By W. F. G. Swann.

In a paper presented at the meeting of the Physical Society on October 11, 1919, the writer showed that the moving line theory of electromagnetic induction was applicable in all cases of moving magnetized bodies provided that, in applying it, one treats the motions of the magnetic lines from the different amperian whirls (or magnetic doublets) of the magnetized substance separately; and, in so doing, takes account only of the rectilinear motions of the whirls, ignoring the rotary motions of the frames of the whirls about their *own* axes. In the case of the pure rectilinear motion of a magnet, the result given by the moving line theory in this form is the same as that which would be given by the ordinary application of that theory, in which the motion of the *resultant* field of all the amperian whirls is discussed.

In the Physical Review for August, 1918, S. J. Barnett describes an experiment which, at first sight, appears to show that, even in the case of the rectilinear motion of a magnet, the moving magnetic lines are not accompanied by an electric field of the type here referred to. The experiment consists in an attempt to charge a condenser by causing an electromagnet to swing past it. The condenser and remainder of the insulated system were, however, shielded by a brass case to protect them from electrostatic influences. The lengths of the magnetic poles parallel to their lines of motion were considerably greater than the corresponding dimension of the case, and the charging effect was sought at the instant when the poles were symmetrically situated with respect to the case, so that, as regards the electric field produced by the moving magnetic lines, the problem may be looked upon as a two-dimensional one, in the yz-plane perpendicular to the line of motion of the magnet.

The object of the paper now presented is to show that the case shields off any effect which might result from the motion of the magnet. For, the moving

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

line theory gives, for the components E_y and E_z of the electric field, in terms of the components H_y and H_z of the magnetic field:

$$E_y = \frac{v}{c} H_z, \qquad E_z = -\frac{v}{c} H_y,$$

where v is the velocity of the magnet, and c the velocity of light.

Now the magnetic field H is derivable from a potential ψ, so that E_y and E_z are also derivable from a potential φ which is, in fact, v/c times the function conjugate to ψ. Now, in the case of any field derivable from a potential, the case will acquire a distribution of charge such as to result in equality of potential not only over its surface but throughout its interior. The field due to the charge distribution on its surface will in fact be such as to exactly annul, at all internal points, the field due to external sources, so that there will be no resultant electric field of any kind within the case. There will consequently be no tendency for the condenser to become charged in such an experiment as that carried out by Barnett.

THE INFLUENCE OF BLOWING PRESSURE ON PITCH OF ORGAN PIPES.[1]

BY ARTHUR C. LUNN.

THE rise of pitch with increasing blowing pressure is a familiar phenomenon but seems to have thus far received no theoretical explanation. It is recognized that a complete theory of the organ pipe offers a complex problem, but the oscillations of the blowing jet are a main feature. It is here suggested that the effect in question is due to the discontinuity of pressure needed to balance the kinetic stiffness of the jet as its curvature alters, and some simple formulas are obtained for the approximate estimation of the rise of pitch in practical cases. The chief result is that the elementary end condition $dy/dx = 0$ is replaced by

$$y - b\frac{dy}{dx} = 0 \text{ at } x = 0, \qquad b = \frac{v^2wh^3}{6q^2Sa},$$

in which v, q, w, h, S, a, are respectively the velocities of sound and of the jet, the width and height of the mouth and cross-section of the pipe, and the effective thickness of the jet or wind-way.

UNIVERSITY OF CHICAGO,
December 4, 1919.

A PHOTOGRAPHIC STUDY OF EXPLOSIONS IN GASES.[1]

BY JOHN B. DUTCHER.

THE apparatus used in this experiment is practically the same as used by Foley in photographing sound waves, with the exception that at the sound spark there is a device for blowing with a mixture of explosive gases a

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 31, 1919.

soap bubble, with the sound spark gap approximately in the center of the bubble. The sound spark generates a sound wave and explodes the bubble of gas. Photographs were obtained of the sound wave due to the spark, both inside and outside of the bubble. The explosion in the gas appears to take place from nuclei, and its character to depend on whether the exploding spark is fat or lean. The explosion surface is so irregular that well defined sound waves are not produced by it.

A Photographic Study of Sound Pulses through Crooked and Curved Tubes, with Deductions Concerning Telephone Mouthpieces, Phonograph Horns, etc.[1]

By Arthur L. Foley.[1]

METAL plates were curved and so placed that the space between them has the shape of a longitudinal section through a telephone mouthpiece, megaphone, etc. These plates were disposed radially about an electric spark and longitudinally around the light rays from the illuminating spark, using the apparatus for photographing sound waves described in a previous paper. In this position the shadow of the metal plates on the dry plate corresponds to a longitudinal section of the mouthpiece, etc. The shadow of the sound wave was photographed for various time intervals, and therefore for various positions, while passing between the plates. A study of the reflected waves shows that the energy reflected at the open end of an organ pipe is small and that the action of ear trumpets and other sound receiving devices is not what it is popularly supposed to be.

The Performance of Conical Horns.[1]

By G. W. Stewart.

THIS is a report upon an experimental study of the action of a conical horn as a receiver. The experiments were performed with an approximate freedom from errors due to reflection, but under conditions that clearly give approximations to the case of a conical horn closed at the vertex.

The results lead to the following conclusions:

1. There is amplification for all frequencies higher than that for fundamental resonance.

2. For each frequency there is an optimum angle which gives the greatest fundamental resonance amplification, the greater frequency requiring the larger angle.

3. This optimum angle depends upon whether the given frequency is a fundamental or an overtone, the angle decreasing with the order of overtone.

4. The optimum angle is less markedly an optimum the higher the order of overtone.

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 31, 1919.

These experimental results are considered in the light of existing theory, especially the quantitative theory of A. G. Webster.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA.

A PHOTOGRAPHIC STUDY OF THE WAVE-FORM OF SOUNDS FROM LARGE GUNS.[1]

BY DAYTON C. MILLER.

BY means of the author's phonodeik, in a newly designed, portable form, photographs of the wave-form of the sounds from large guns were made at Sandy Hook Proving Ground, in 1918 and 1919. Records were obtained from 6-inch, 10-inch, 12-inch and 14-inch rifles, at distances varying from a hundred feet to three thousand feet from the gun, and from stations both in front of and behind the gun.

All of the photographs show the same general characteristics. There is no true vibratory wave form; there is a single positive (compression) pulse, rising rather abruptly to a maximum, and falling to a negative value (rarefaction). The numerical value of the negative pressure is usually much less that that of the positive pressure, but it lasts for a longer time. On the return of the pressure to the normal value there is some fluctuation which shows a billowly action having a kind of irregular vibratory motion.

The photographs show the sounds which originate in two distinct sources; one the disturbance produced by the projectile itself (called the "crack") as it moves with a velocity of about 2,600 feet per second, and the other, the sound coming from the muzzle of the gun (the "boom" or "muzzle wave") which, of course, moves with the velocity of sound, 1,100± feet per second. (A study of the velocity of the pressure-wave near the muzzle of the gun, and to distances of several miles, is the subject of a report to be made later.)

The record of the sound waves from a 14-inch rifle, taken at a station about 3,000 feet in front of the gun, shows the "crack" due to the projectile as it passes the station at a distance of about 700 feet, followed about one second later by the "boom" from the gun. The crack consists of a sharp, positive pulse followed quickly by a negative pulse of equal value and by a second positive pulse, the three pulses together lasting about 2/100 second. After this the disturbance subsides until the arrival of the boom. The boom consists of a positive pulse, the maximum pressure of which is equivalent to about 1/400 atmosphere, the duration being about 2/100 second, followed by a negative pressure of about one-fourth the value of the positive and continuing for about 1/10 seconds.

CASE SCHOOL OF APPLIED SCIENCE,
December 20, 1919.

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 31, 1919.

The Calibration of a Sound Chamber and Sound Sources and the Measurement of Sound Transmission of Simple Partitions.[1]

By Paul E. Sabine.

In making acoustical measurements, in which the unit of sound intensity for any frequency is the minimum audible intensity for that frequency, there must be known the absolute rate of decay of sound in a room whose acoustic properties may be maintained constant, and also the rate of emission of sound in a corresponding unit, of such standard sources of sound as are to be used. The arrangement of the new Wallace Clement Sabine Laboratory for securing the best conditions for such measurements is described. The measurement of the absolute rate of decay in the sound chamber of the laboratory, together with the rate of emission of sound by the pipes of a complete organ of seventy-two pipes is outlined.

Some of the factors entering into the apparent transmission of sound through wall partitions are considered. The method of measuring the effective coefficient of transmission is discussed and numerical data for a number of simple constructions are given.

Wallace Clement Sabine Laboratory, Geneva, Ill.

Transmission of Sound Through Walls.[1]

By F. R. Watson.

THIS is a continuation of the earlier investigations of transmission of sound through materials,[2] the measurements being extended in this case to plaster walls of some thickness. The amount of sound transmitted through such walls is very small so that special arrangements of apparatus were required to make certain that the sound came through the wall under test and that any possibility of leakage of sound was minimized as far as possible. The experiments were conducted in two basement rooms with double brick walls and air space. The doorway connecting the rooms was closed by three sound-proof doors. The whistle generating the sound of 512 vibrations per second was blown by a constant air pressure. It was placed at the focus of a parabolic reflector so that practically all the sound was directed initially to the wall under test. After reflection, the sound was absorbed rapidly by heavy padding. The transmitted sound was measured quantitatively by a Rayleigh resonator specially designed to be extremely sensitive. Comparative measurements were taken on various types of walls. The influence of cracks and apertures in transmitting sound was investigated as well as the tuning of these partitions by applied pressure.

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 31, 1919.

[2] Phys. Rev., Vol. 7, pp. 125 and 287, 1916.

CHARCOAL ABSORPTION AND CYCLIC CHANGES.[1]

BY THOS. E. DOUBT.

THIS is a continuation of the work reported at the meeting of November, 1919.

The absorption of gases as well as their evolution by activated charcoal are considered as processes in a cyclic operation. Data and curves are given. It is shown that the process is accurately reversible. Heat is evolved in absorption and required in evolution. Experiments have been carried out for the determination of the amount of heat involved.

The amount of work involved in the operation may be readily calculated from the data given. The laws of thermodynamics become applicable as well as the phase rule. The condition at any instant can be represented by an equation similar to that for a vapor.

THE HEAT OF VAPORIZATION AND WORK OF IONIZATION.[1]

BY C. S. FAZEL.

THE work necessary to separate atoms and electrons may be used to correlate the atomic, chemical and electrical forces. There are four types of work which may be considered. They are the work necessary to: (1) Separate two neutral atoms, (2) take a neutral atom from a solid, (3) remove an electron from a solid, (4) remove an electron from from an atom in the vapor state.

The work required to separate two neutral atoms may be obtained from the heat of dissociation of the element per molecule.

The latent heat of vaporization per molecule gives the work necessary to separate a neutral atom from a solid.

The work necessary to separate the electron from a solid is given either by the long wave-length limit in the photoelectric effect or from the latent heat of vaporization of electrons in the thermionic effect.

The ionization potentials of the metals give a method of measuring the work necessary to remove an electron from a neutral atom in the vapor state.

These four types of work are calculated for various elements and compared.

ENERGY CONTENT OF CHARACTERISTIC RADIATIONS.[1]

BY CHESTER W. RICE.

THE following empirical equation for the energy content of characteristic radiations may possibly be of interest due to its apparent wide generality.

$$H = AmI\,(V - V_0)^2\,N^2hC\left(\frac{1}{\tau^2} - \frac{1}{n^2}\right)\epsilon^{-(k/N)\sqrt[3]{(\rho/M)}\,n^2}, \qquad (1)$$

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, January 1, 1920.

where:

A = constant for x-ray tube and arrangement of apparatus, depending upon width of slits, angle of viewing, form of focal spot, dimensions of anode, etc.

m = number of atoms under observation of the kind being considered.

I = electron current density.

V = potential difference across tube.

V_0 = critical potential below which characteristic lines are not obtained.

N = atomic number of material under consideration at anode.

h = Planks' constant.

C = Rydberg constant.

τ = 1, 2, 3, 4, etc., depending on series under consideration, $\tau = 1$ for K series, $\tau = 2$ for L series, $\tau = 3$ for M series, etc.

n = term number for series under consideration.

k = constant for all the elements.

ρ = density of radiator in grams per c.c.

M = atomic weight of anode or radiator.

The general method of arriving at the above equation is as follows:

The energy radiated per atom is assumed to be that given by the quantum relation.

$$W = h\nu, \tag{2}$$

where

ν = frequency of radiation in cycles per second.

The frequency of characteristic radiations, as far as first order effect are concerned, is assume to be given by the familiar equation

$$\nu = N^2 C \left(\frac{1}{\tau^2} - \frac{1}{n^2} \right), \tag{3}$$

from which we obtain by combining (2) and (3).

$$W = N^2 h C \left(\frac{1}{\tau^2} - \frac{1}{n^2} \right), \tag{4}$$

as the energy radiated per atom.

If we now multiply (4) by the probability that a radiating atom will give off energy of this frequency, or, what is the same thing, the proportion of radiating atoms which are emitting this frequency, we should obtain an expression for the relative energy content of the characteristic lines.

A study of some of the work done on the hydrogen spectrum[1] as well as a study of x-ray data, led me to write the probability factor

$$P = \epsilon^{-(k/N)\sqrt[3]{(\rho/M)}\, n^2}, \tag{5}$$

From (4) and (5) we obtain for the energy of a particular frequency reaching

[1] Evershed, Phil. Trans. Roy. Soc., Vol. 201, 1903. Mitchell, Astro. Jour., Vol. 38, 1913, p. 407. Dyson, Proc. Roy. Soc., Vol. LXVIII, 1901, p. 35, and Phil. Trans., Vol. 206, 1906. J. Holtsmark, Annalen der Physik, Vol. 55, 4th Series, Heft 4, 1916, pp. 245–298. Beatyt, Phil. Mag., Vol. 33, 1917, p. 49.

the measuring device

$$H = Km\ N^2hC\left(\frac{1}{\tau^2} - \frac{1}{n^2}\right)\epsilon^{-(k/N)\sqrt[3]{(\rho/M)}\,n^2}, \tag{6}$$

where K = constant depending upon excitation conditions and the fraction of total energy reaching the measuring device.

For x-ray tubes some of the factors entering into the constant K may be inserted.

Theoretical work by Davis[1] and experiments by Wooten[2] indicate that the intensity of the lines in an x-ray tube is proportional to $(V - V_0)^2$ for constant electron current.

As far as order effects are concerned we may write

$$V_0 = N^2\frac{hC}{e}\left(\frac{1}{\tau^2} - \frac{1}{\infty^2}\right), \tag{7}$$

where e = electron charge.

We thus have been led to equation (1) as expressing an empirical relation, for the energy content of characteristic x-ray spectral lines as far as first order effects are concerned.

In the K x-ray series, we obtain, from equation (6) assuming constant excitation conditions,

$$\frac{H_\alpha}{H_\beta} = \frac{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)\epsilon^{-(4k/N)\sqrt[3]{(\rho/M)}}}{\left(\frac{1}{1^2} - \frac{1}{3^2}\right)\epsilon^{-(9k/N)\sqrt[3]{(\rho/M)}}} = .845\epsilon^{(5k/N)\sqrt[3]{(\rho/M)}}. \tag{8}$$

In the same manner we may write down the intensity ratios between various lines in the same series or obtain relations between the intensities of lines in different series.

Below I have calculated the value of k from the best available data assuming equation (8).

Element.	N.	M.	σ.	H_α/H_β.	k.	Data by
Mo.....	42	96	9.0	5.55	34.0	Wooten[2]
Rh.....	45	102.9	12.1	6.3	37.0	Webster[3]
Pd.....	46	106.7	11.4	6.25	38.8	Wooten[4]

The value of the constant k from the date given by Merton and Nicholson[4] for the Balmer series of hydrogen, assuming that their discharge tubes were filled with H_2 at a pressure of .76 mm. of Hg or 10^3 bars and 0° C.

Since,

$$\rho = .0893 \times 10^{-9}p, \tag{9}$$

for $p = 10^3$ bars and 0° C., $\rho = .0893 \times 10^{-6}$ grams per c.c.

[1] Bergen Davis, PHYS. REV., June, 1918, p. 433.

[2] B. A. Wooten, PHYS. REV., Vol. XIII, No. 1, Jan., 1919, p. 71.

[3] Webster, PHYS. REV., Vol. VII, No. 6, 1916, p. 607.

[4] Merton and Nicholson, Phil. Trans. Roy. Soc., Series A, Vol. 217, 1917, p. 260.

We obtain

$$H_\alpha/H_\beta = 3.83, \text{ giving } k = 52.7,$$

$$H_\beta/H_\gamma = 1.43, \text{ giving } k = 11.8,$$

$$H_\gamma/H_\delta = 1.67, \text{ giving } k = 11.7.$$

It seems very probable that the pressure decreased considerably during the time of exposure due to clean up. This would mean a decrease in intensity ratios as time went on.

In Table I. are given some calculated data on the K and L x-ray series, for some of the elements in the solid state, assuming $k = 37$.

TABLE I.

Element	Solid N.	M.	Grams per c.c. ρ.	K Series. H_α/H_β.	H_β/H_γ.	H_γ/H_δ.	H_α/H_γ.	H_α/H_δ.	L Series. H_α/H_β.	H_β/H_γ.
H_1 ...	1	1.008	.07	10^{33}	— —	— —	— —	— —	— —	— —
C....	6	12.00	2.3	2×10^{7}	7×10^{10}	8×10^{13}	4×10^{18}	— —	5.5×10^{10}	— —
Na...	11	23.0	.97	169.	3800	34,000	1×10^{6}	— —	2980	31,200
K....	19	39.1	.88	13.2	44.6	138	593	— —	34.9	126
Ni ...	28	58.68	8.9	29.1	134	585	3930	— —	105	535
Cu...	29	63.57	8.9	23.1	96.5	374	2240	— —	75.5	342
As ...	33	74.96	5.73	24.8	26.6	71.	245.	— —	20.8	65.
Se....	34	79.20	4.80	19.3	19.0	45.	134.	5860	14.9	41.2
Rb...	37	85.45	1.53	3.1	5.9	10.2	18.4	188	4.6	9.3
Mo...	42	96.00	9.0	6.3	15.8	36.4	100.	3600	12.4	33.3
Ru...	44	101.70	12.3	6.8	17.3	40.6	112.	4700	13.5	37.2
Rh...	45	102.9	12.1	6.3	15.9	36.8	103.	3720	12.4	33.7
Pd...	46	106.7	11.4	5.7	13.7	30.2	78.	2330	10.7	27.6
W....	74	184.0	19.3	2.8	4.9	8.1	13.6	110	3.8	7.4
Pt....	78	195.2	21.7	2.6	4.8	7.6	12.4	95	3.8	7.0
U_1....	92	238.2	18.7	2.0	3.1	4.6	6.3	29	2.4	4.2

Inspection of Table I. indicates that δ lines might be detected in *Rb*, *W*, *Pt*, if carefully looked for without resorting to dilution.

In the gaseous state, we may obtain low densities of the element under observation and therefore have an opportunity of obtaining more lines.

The possibility of diluting the element to be studied by alloying, deserves consideration, especially, if an alloying metal can be found which gives low intensity general radiation at the voltage necessary for good characteristic radiation.

As a rough check on the equations as applied to alloys, I have calculated below, the values of density ρ' required to give the intensity ratios observed by Webster[1] for the Ru impurity in his Rh target.

H_α/H_β Calculated for Normal ρ.	H_α/H_β Observed by Webster.	Required ρ.[1]
6.8	3.5	$.14\rho$
17.3	4.0	$.012\rho$

[1] Webster, PHYS. REV., Vol. VII, No. 6, 1916, p. 607.

Another method of making the same estimate is to compare the intensity of the α line for Rh with the α line for Ru. This procedure seems legitimate in this case, since the two materials are incorporated in the same target, giving constant excitation conditions, etc. Under these conditions, we may also probably assume that the number of atoms m of the kind under consideration is inversely proportional to the mean atomic volume.

$$\frac{m \text{ of target}}{m \text{ of impurity}} \propto \frac{\frac{\rho_t}{M_t}}{\frac{\rho_i}{M_i}}. \tag{10}$$

In this case we will make the approximation

$$\rho_t = \rho_{Rh} \text{ and } M_t = M_{Rh}$$

which is of course only legitimate when dealing with small impurities.

$$\frac{H_{Rh_\alpha}}{H_{Ru_\alpha}} = \frac{\frac{\rho_t}{M_t}(V - 23.2)^2(45)^2\epsilon^{-(4k/45)\sqrt[3]{(\rho_t/M_t)}}}{\frac{\rho_i}{M_i}(V - 22.2)^2(44)^2\epsilon^{-(4k/44)\sqrt[3]{(\rho_i/M_i)}}} \tag{11}$$

for $V = 40$ kv. and assuming $k = 40$,

$\rho_{Rh} = 12.1$ ρ_i = density of *Ru* impurity,

$M_{Rh} = 102.9$ $M_i = M_{Ru} = 101.7$.

From Webster's[1] data we estimate $H_{Rh_\alpha}/H_{Ru_\alpha} = 35.5$, which gives,

$$\log \rho_i = -2.885 + .786 \sqrt[3]{\rho_i}, \tag{12}$$

from which by trial

$$\rho_i = .08 \text{ grams per c.c.}$$

The two methods of estimating the amount of Ruthenium impurity in Webster's Rhodium target thus lead to results of the same order of magnitude.

Here we should observe that Holtsmark[2] found that the presence of iodine, oxygen, argon and helium had no effect on the relative intensity of the hydrogen lines under observation. The relative intensity merely depending upon the density of the hydrogen. This work indicates that the crowding effect is independent of the proximity of unlike atoms. The enhancement of the hydrogen lines in the presence of Neon as observed by Merton and Nicholson[3] would appear to show that there are exceptions.

From data on the intensity of the hydrogen lines in the sun's chromosphere we estimate the exponent of equation (6) to be approximately $.00367\, n^2$.

Thus

$$\frac{k}{N}\sqrt[3]{\frac{\rho}{M}}n^2 = .00367\, n^2. \tag{13}$$

[1] Webster, PHYS. REV., Vol. VII, No. 6, 1916, p. 607.

[2] See note 1, p. 233.

[3] Merton and Nicholson, Phil. Trans. Soc., Series A, Vol. 217, 1917, p. 260.

If $k = 40$ $N = 1$ $M = 1.008$, $\rho = 770. \times 10^{-15}$ grams per c.c. density of H_1 in sun's chromosphere. This corresponds to what is ordinarily considered to be an extremely good vacuum.

It is interesting to note that the form of this function assuming n to be a continuous variable, is very strikingly similar to the energy wave-length distribution curves obtained for the continuous X radiation, as given for example, by the recent work of Ulrey.[1] A discussion of this application, will, however, have to be postponed until a later date.

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.,
December, 1919.

THE ZEEMAN EFFECT FOR ELECTRIC FURNACE SPECTRA.[2]

BY ARTHUR S. KING.

THE experiments were made with a simple form of tube resistance furnace placed axially between the poles of a large Weiss electro-magnet, by which, with a gap of 10 cm., a field of about 6,500 gausses was obtained. The furnace was operated at atmospheric pressure, the jacket around the tube, as well as the contact pieces at the ends, being water-cooled. The observations thus far made are for the n components only, the vapor in the tube being viewed along the lines of force. Both emission and absorption spectra were thus photographed, the latter being produced by placing a graphite plug in the hottest part of the tube.

An extended series of observations, chiefly for the spectra of iron and vanadium, showed the characteristics of the Zeeman effect for this source and the special advantages possessed by the furnace for the study of certain types of lines. It has previously been shown that many lines, faint in the arc, are strong in the furnace. Such lines are usually obtained only with extreme difficulty when the spark, the usual source for magnetic field experiments, is employed. The magnetic separations of many such lines were measured and form a considerable addition to the material obtainable with the spark. In the iron spectrum, lines of this type as a rule are of wide separation, and the sharpness of the components in the low field used indicated a simple triplet structure. These lines as a class are strong in sun-spot spectra.

Lines common to the furnace and the spark showed no difference either in the number of their n components or in their separation when the same field was used for both sources. The fact that no alterations of magnetic type result from the very different excitation in the furnace and spark permit either to be used for the sort of lines best produced by it. A combination of high magnetic field, the apparatus for which is being prepared, with a sharpening of the components through enclosure of the furnace in a vacuum chamber will furnish measurements of high precision.

[1] Ulrey, PHYS. REV., Vol. XI., No. 5, May, 1918, p. 407.

[2] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, January 1, 1920.

The "inverse" Zeeman effect, in which the magnetic components appear as absorption lines, was studied by the use of a plug in the furnace tube. This reversed the spectrum and the absorption components were found the same in character as those given in emission. The study of absorption lines in the magnetic field by this method may be expected to be of much service in connection with solar phenomena.

MOUNT WILSON OBSERVATORY,
November 26, 1919.

CRITICAL POTENTIALS OF THE "L" SERIES OF PLATINUM.[1]

BY DAVID L. WEBSTER.

THIS work is a continuation of an investigation begun by Dr. H. Clark and the author and on which a preliminary report was published in the Proceedings of National Academy, March, 1916. That work indicated that the "L" series must be divided into at least two and probably three subseries. The first of these, called L_1, found to contain at least the lines α_1 and β_2. (Siegbahn's Notation).

The L_2 series contained at least the lines β_4, β_1, and γ_1 and the L_3 series should probably contain γ_4 and perhaps one or two others.

The present work was done by a photographic method with a modified form of Hull's constant potential outfit. The principal improvement in this machine consisted in the use of a 500-cycle generator, separate from the motor, with its field excited by battery current, and a variable condenser across the secondary of the 500-cycle transformer.

When this condenser was properly adjusted at a point near the value giving resonance the changes of speed due to fluctuations of voltage on the motor could be made to produce only small second order effects on the final D.C. voltage. With this equipment and a mica spectrograph the critical potentials were determined for the rest of the lines found by Siegbahn and Friman in platinum.

The L_1 series was found to contain the lines l, α_2, α_1, β_2, β_5 and a new line called β_6. The L_2 series contains η, β_4, β_1, γ_1, γ_2 and perhaps β_3. The L_3 series contains γ_4, γ_3 and perhaps β_3. The critical potentials of the L_3 lines are not accurately known, and there are theoretical reasons for believing that they are not all equal.

The critical absorption frequency corresponding to the L_1 series coincides approximately with that of β_5 and that corresponding to the L_2 series lies between γ_2 and γ_3.

The line β_6 probably corresponds to Dershem's tungsten line at 1.220Å. The present apparatus was not sensitive enough to show the fainter lines found by him and by Overn in tungsten.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
CAMBRIDGE, MASS.

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, January 1, 1920.

THE SPECTRUM OF RADIUM EMANATION.[1]

BY R. E. NYSWANDER, S. C. LIND AND R. B. MOORE.

THE spectrum of radium emanation has been photographed between the limits λ3982 and λ7449 with a four-prism spectrograph having a dispersion of 13 A.U. per millimeter at λ4000 and 136 A.U. at λ7000. The spectrum of helium was used as a comparison standard and the emanation wave-lengths computed by the use of Hartmann's dispersion formula. The spectra were also observed visually with a Hilger constant deviation spectrometer. Forty lines were measured of which nineteen, including the most prominent ones, are in excellent agreement with the results of Rutherford and Royds and Watson. Five new lines were photographed in the red end of the spectrum and a group of ten faint lines, nine of which have not been previously recorded, appeared between the limits λ4510 and λ4463 inclusive. This group, however, appeared only in one photograph. The remaining new lines were distributed through the spectrum. Varying amounts of emanation from 200 millicuries to about 1,000 millicuries were employed.

The emanation was purified by the method of Duane[2] and in addition it was frozen down in liquid air for removal of residual gases by pumping. The gas was collected in small glass vaccum tubes of capillary bore with platinum electrodes sealed in very small bulbs at the ends. The tubes were excited with a medium sized induction coil, exposures varying from 30 minutes up to two hours. No difficulty was experienced from the driving of the emanation into the walls of the tube. The occlusion thus produced in most cases was very small; in one tube, after several hours excitation, the occluded gas was less than one per cent., and in one tube only was it as high as 35 per cent.

ON THE POSSIBILITY OF PULLING ELECTRONS FROM METALS BY POWERFUL ELECTRIC FIELDS.[1]

BY R. A. MILLIKAN AND B. E. SHACKELFORD.

DURING the past ten years the first of the authors has made a long series of experiments upon this subject with the following general results. In a vacuum so high that the gas plays no further rôle in the discharge (the measured pressures were of the order 10^{-6} mm. of mercury) the potential gradients at a metal surface necessary to obtain a discharge are exceedingly variable even from the same surface. With bright, clean metal surfaces which have not been especially treated with a view to removing occluded gases and other surface impurities the potential gradient necessary to start a minute leak such as is detectable with a delicate tilted gold-leaf electroscope is in general between 10,000 and 50,000 volts per mm. and may be controlled within limits by various expedients to be described in detail in the complete paper.

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, January 1, 1920.

[2] W. Duane, PHYS. REV., V., p. 311, 1915.

By heating tungsten electrodes to a red heat in a specially constructed tube designed by the first of the authors we have found that the potential gradient below which no discharge whatever takes place may be raised to from 40,000 to 70,000 volts per mm. By heating the tungsten electrodes to 2700° K. we put them into condition so that no leak whatever could be produced with potential gradients lower than 430,000 volts per mm., and the first spark occurred at 600,000 volts per mm. or *six million volts per centimeter*. These experiments indicate that the discharge from the surface of a metal in a perfect vacuum is conditioned by surface impurities of some sort and cast doubt upon the conclusion that there is a particular field strength at which electrons begin to be pulled out of a pure metal.

THE DEFLECTION OF ALPHA PARTICLES THROUGH LARGE ANGLES BY LIGHT ATOMS.[1]

BY LEONARD B. LOEB.

IN a recent paper[2] Prof. Rutherford showed that the number of light atoms (*e.g.*, H, N, O) thrown straight forward with a maximum velocity when struck by swift alpha particles is many times greater than the number to be expected on the simple theory of scattering of Darwin.[3] At the suggestion of Prof. Rutherford an attempt was made to corroborate these results by counting the number of scintillations produced by alpha particles deflected through large angles after impact with light atoms whose mass was greater than that of the alpha particles. The problem was found practically insoluble except for aluminium due to experimental complications introduced by; the small number of alpha particles deflected through large angles, the short ranges of the recoiling alpha particles, the intense gamma radiation from the powerful sources of Ra C used, and the contamination due to the diffusion of radio-active material from the source. The number of particles observed under given conditions of experiment were compared with the number computed, assuming the same conditions as nearly as possible and using Darwin's law of scattering. The ratio of the number observed to the number computed for particles deflected through 105° was determined for the following elements, Pb, Au, Ag, Cu, S and Al. They are given in the table below together with the nearest distance of approach of alpha particles and nucleii, apsidal distance, under the conditions of the experiment. The ratio is less than unity because of certain geometrical arrangements used in practice, which made exact computation difficult. It may be seen that for all cases where the nearest distance of approach is greater than 7×10^{-13} cm., the ratio is nearly constant. For aluminium and possibly for sulfur it appears to be greater, suggesting an abnormal deflection of the particles as predicted by the experiments of Prof. Rutherford. Due to the small numbers of particles counted per minute, too great a confidence cannot be however be placed in these results.

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, January 1, 1920.

Substance.	Al.	S.	Cu.	Ag.	Au.	Pb.
No. obs. per 10 mins. per 10 Mg. RaC..	1.44	1.83	5.67	17.7	31.6	32.3
Ratio observed to theory	.613	.497	.476	.484	.475	.466
Apsidal distance $\times 10^{-13}$ cm.....	6.96	8.43	14.6	26.5	38.4	39.8

THE CONSTRUCTION AND CALIBRATION OF A DEVICE PERMITTING THE APPLICATION OF A CURRENT PULSE FOR A PRE-DETERMINED NUMBER OF MILLISECONDS.[1]

BY LINDLEY PYLE.

A SPRING-driven device drives a small brush at high speed across the face of a triangular strip of copper. Variation of the location of the line of passage from a region near the base of the triangle to a region near the apex permits a variation in the duration of an accompanying current flow. Calibration in known time intervals may be made by incorporating the make-and-break mechanism into a condenser circuit. The results may be checked by oscillographic data.

THE INFLUENCE OF AIR VELOCITY AND ANGLE OF INCIDENCE ON THE DISSIPATION OF HEAT.[1]

BY T. S. TAYLOR.

WHILE conducting some experiments to secure useful data on the dissipation of heat from surfaces such as are found in electrical machinery, some rather interesting fundamental results were obtained. The results herein contained were determined from data obtained while making tests on the dissipation of heat from the surfaces of the end coils of turbo generators. A heater, having the same cross section as the end coils, *i.e.*, about 4 cm. square, and a length of 50 cm., was constructed, and covered with the typical treated cloth wrapper that is used on such end coils. The temperatures just under the wrapper and on the outside surface of the same were measured by means of thermocouples made from 0.13 mm. copper and constantan wire. The heater was placed in a current of air which could be given various velocities from 0 to 100 meters per min. Observations were taken of the internal, surface, and room temperatures for various inputs and air velcoities. The distribution of the velocity in the air stream was determined by means of a Pitot tube made from hypodermic needles.

For a definite input or dissipation per unit area, the temperature excess of the surface above that of the temperature of the air stream was found to vary approximately inversely with the velocity. The watts dissipated per unit area varied directly as the temperature excess for each constant air velocity.

A very marked difference was found between the amounts of heat dissipated

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

when the air was blowing at right angles and when at an angle of 45°. For an air velocity of 100 meters (4,000 ft.) per min. and a definite input, the temperature excess of the surface of the heater above air temperature when the heater was at right angles to the air stream was found to be 40 per cent. more than when it was at an angle of 45° to the air stream. A somewhat similar condition was also found to hold for a small coil of wire 3 mm. in diameter.

The thermal conductivity of the material used for such wrappers was found from a large number of determinations to be 0.00030 cal/°C./cm./sec.

REACTIVE HYDROGEN IN THE ELECTRICAL DISCHARGE.[1]

BY GERALD L. WENDT AND ROBERT S. LANDAUER.

SIR J. J. Thomson[2] demonstrated in 1912 by means of positive ray analysis that there exists in the high vacuum electrical discharge in hydrogen a substance with the molecular weight of three, to which he gave the tentative name of X_3. Its reactions left little doubt of its being a triatomic form of hydrogen. In 1915 Dempster,[3] using the electrical rather than the photographic method, showed the presence of H_1, H_2 and H_3 in such a tube, with H_3 predominating at pressures of .01 mm. Duane and Wendt[4] showed in 1916 that hydrogen on exposure to alpha radiation from niton, becomes chemically active, and gave some evidence to show that there is at the same time a contraction in volume, indicating the formation of a molecule larger than H_2. This contraction has since been confirmed by Lind.[5]

When a stream of hydrogen at about 7 cm. pressure is passed through an electrical discharge tube, the hydrogen shows a chemical activity much higher than that of the ordinary gas. On passing the hydrogen through a tube filled with powdered sulfur, hydrogen sulfide is formed, and can be made to record itself by passage over a strip of filter paper moistened with lead acetate solution. The amount of lead sulfide formed indicates the conversion of about one hundredth of one per cent. of the hydrogen into the active form.

The chemical activity of this gas is not due to the presence of ions. If after passing through the electrical discharge, the hydrogen is allowed to pass through a very sensitive electroscope used for the determination of small quantities of radium emanation, the leaf of the electroscope remains stable. The ions formed in the discharge pass no further than the discharge tube, although when no discharge is used the hydrogen may be previously ionized, and will then rapidly discharge the electroscope on passage through it.

That this active form of hydrogen is due to the presence of large molecules, and not to single atoms, is indicated not only by the work cited above, but also by the properties of the monatomic hydrogen prepared by Langmuir. This

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, January 1, 1920.

[2] Proc. Roy. Soc., A, 89, 1, 1913.

[3] Phil. Mag., 31, 438, 1916.

[4] PHYS. REV., 7, 689, 1916.

[5] J. Amer. Chem. Soc., 41, 531, 1919.

form exists only at very low pressures, and is so readily adsorbed by glass that it will not pass through a plug of glass wool. The triatomic form, however, passes through glass wool readily, and is fairly stable at pressures as high as atmospheric. It decomposes spontaneously in the course of several minutes. It is either condensed or decomposed at the temperature of liquid air.

The vacuum discharge is not needed to produce this gas. The corona discharge from an alternating current 60-cycle transformer, at 20,000 volts in in the purest hydrogen, at atmospheric pressure imparts a similar reactivity to the hydrogen, although the proportion transformed by this means is somewhat smaller.

UNIVERSITY OF CHICAGO,
CHICAGO, ILLINOIS.

AN UNDAMPED WAVE METHOD OF DETERMINING DIELECTRIC CONSTANTS OF LIQUIDS.[1]

BY W. H. HYSLOP AND A. P. CARMAN.

THE method makes use of two circuits of the "ultraudion" type, one operating at a definite frequency and the other tuned to such a frequency that a definite number of beats per second is obtained. The accompanying sketch shows one arrangement of the circuits. Circuit I., very loosely coupled with Circuit II., is operated at one frequency throughout a test. Circuit II., shown in sketch, differs from Circuit I. in the use of three condensers

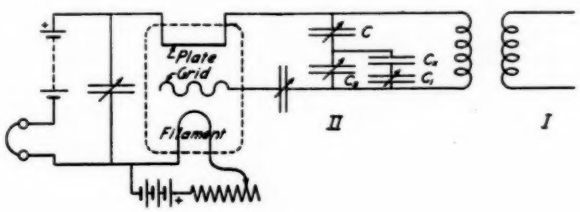

Fig. 1.

C, C_s and C_x, instead of a single condenser. The condenser C is inserted in order to keep the resultant capacity of the group at the proper value. The condensers C_s and C_1 are air condensers of the Korda type. They were designed and built in the department machine shop especially for this work, and can be reset to any required capacity with high accuracy. The calibration also showed that the capacity from the minimum to the maximum values varies directly as the angular displacement of the plates. The test condenser C_x consists of two concentric cylinders, and its value is known in terms of the angular displacement of the plates of C_s.

In determining the dielectric constant of a given liquid, the condenser C_x is disconnected, C_s is set to some value near its maximum, and C is adjusted

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 30, 1919.

until a beat note of convenient frequency (512) is obtained. This beat note is determined by ear through comparison with a standard tuning fork. C_x is filled and connected in the circuit; C_s is decreased until the same beat note is heard. The ratio of this angular displacement to that for C_x as an air condenser gives the dielectric constant of the substance tested. For values beyond the ordinary range of C_s, the condenser C_1 is inserted in series with C_x.

Preliminary measurements for a number of liquids give the following results for a wave-length of approximately 5000 meters, and a temperature of 21° C.

Substance	Diel. Const.
Kerosene	2.12
Toluol	2.39
Ether (abs. ethyl)	4.32
Olive oil	3.11 (a good commercial grade)
Petrol ether	1.86 (boiling point 50–60° C.)

The readings are accurate to about 1 per cent. Further refinements in determining condenser settings will increase the accuracy of the results. The method is easy and rapid of manipulation, and its accuracy can beincreased to any desired extent by using variable condensers of sufficient accuracy.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
December 13, 1919.

A PHOTOGRAPHIC METHOD OF MEASURING THE INSTANTANEOUS VELOCITY SOUND WAVES AT POINTS NEAR THE SOURCE.[1]

BY ARTHUR L. FOLEY.

THE apparatus used in this experiment is the same as described by the author in a paper published in Volume XXXV. of the PHYSICAL REVIEW under the title "A New Method of Photographing Sound Waves," with the following modifications:

Parallel with and just in front of the photographic dry plate was placed a toothed steel disk capable of being rotated at a high and determinable speed. The edge of the disk was near one edge of the dry plate, thus leaving the greater part of the dry plate for the sound wave image. The interval between the sound and light sparks, each of which registers on the dry plate a certain position of the teeth of the disk, was determined by measuring the angle through which the disk rotated in that interval. The radius of the sound wave was calculated from its image on the dry plate. Obersvations were made for various time intervals and sound wave radii and the results plotted. The tangent to the curve at any point gives the instantaneous velocity of sound at that point. Measurements were made for sound waves ranging in radii from .2 centimeter to 50 centimeters. The velocity at points very near the source was found to be two or three times as great as the velocity at points only 2 cm. from the source.

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, January 1, 1920.

A POSSIBLE STANDARD OF SOUND. I., STUDY OF OPERATING CONDITIONS; II., STUDY OF WAVE-FORM.[1]

BY CHAS. T. KNIPP.

I. The dimensions of the several branches of the recently described singing tube,[2] while admitting of some variation, are, in a sense, critical. The tube to sound its fundamental with *ease* must have definite lengths as regards the various vibrating columns, and also definite cross-sectional areas. The usual geometrical construction for standing waves in air columns greatly aids in anticipating the former. Indeed, the form of the sounding tube may be modified or even changed entirely, yet if its dimensions conform to the construction for nodes and loops it may sound, however, not necessarily, its fundamental. A striking example is given in Figs. 1 and 2. Fig. 1 represents the

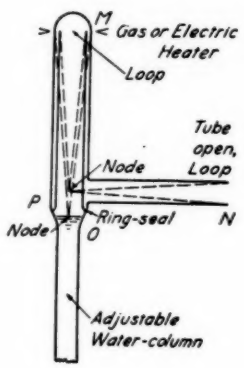

Fig. 1.

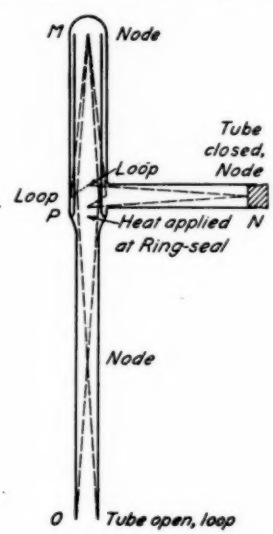

Fig. 2.

typical form. The heat is applied at M by means of a ring gas burner or an electric heating coil. A loop is, therefore, formed at this point and the tube sounds its fundamental, there being a loop at N and a node at O, also one at P midway between M and N.

If now the tube be modified as shown in Fig. 2, making M a node, and P a loop, then the construction gives *two* loops superposed at the ring-seal P. With this arrangement the tube should sound provided the air within can be set in vibration. This may readily be done by applying heat energy at the ring-seal, and by so doing energy is supplied at *two* loops simultaneously, and there should result a tone of double intensity, provided the heated portions are at exact loops.

[1] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, December 31, 1919.

[2] PHYS. REV., N. S., XII., No. 6, December, 1918.

The above construction is readily verified by experiment.

Other constructions that will give forth tones may thus be anticipated. Several tubes illustrating the above points were exhibited and their sound producing qualities demonstrated.

II. The wave-form of the tone emitted was photographed recently in our laboratory by Mr. C. J. Lapp by means of a vibrating membrane carrying a delicately mounted mirror. The light from a ninety degree arc lamp was focussed on the mirror and in turn upon the moving film by a combination

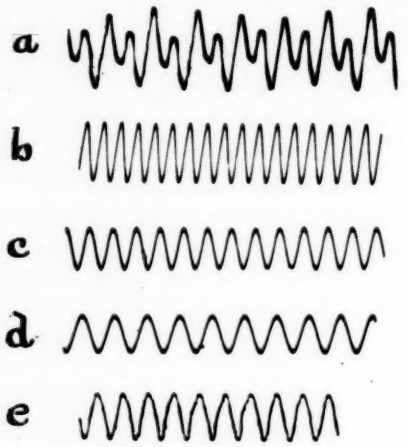

Fig. 3.

spectacle and cylindrical lens system. The inertia of the vibrating diaphragm was very small. The instrument was used without a receiving horn. The system responded freely to overtones not far removed from the fundamental. This was tested by sounding numerous known combinations, and was also shown by blowing an organ pipe to give audible overtones. The photograph of the wave-form is shown at *a*, Fig. 3.

Photographs were shown exhibiting the wave-form under a variety of conditions, the more important of which are: (1) two similar tubes (Fig. 1) were

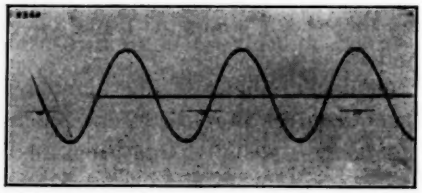

Fig. 4.

adjusted to produce three or four beats per second; and (2) superposed upon this an organ pipe sounding a tone nearly an octave above.

A number of exposures also were made by Mr. Lapp with the straight

form of singing tube.[1] The conditions under which the tube was sounded were, (1) no extension, but the resonator attached direct, the corresponding wave-form is shown at *b*, Fig. 3; (2) an extension but no resonator, *c*; (3) with a longer extension and no resonator, *d*; finally, (4) an L-form of tube, as shown in Fig. 1, with water column at the ring-seal, no extension or resonator, *e*. These four wave-forms are remarkably smooth. This may in part be due to a lack of sensitiveness of the vibrating system, however, *a* in Fig. 3, as explained above, shows that the system must have been fairly sensitive to overtones.

In this study valuable assistance was received from Professor D. C. Miller who kindly offered to photograph the wave-form by means of his phonodiek. Two of the photographs made, using the L-form of tube, are reproduced in Figs. 4 and 5. In Fig. 4 the top of the water column (see Fig. 1) was 11.2 cm. below the ring-seal, while in Fig. 5 the water level was 16.2 cm. below. Fig. 4

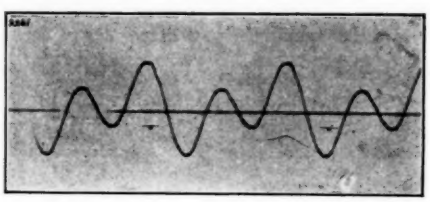

Fig. 5.

is quite accurately a sine curve, excepting a slight indication of a higher octave near the crest just to the right. Fig. 5 also shows the same tendency. The slight non-conformity in Fig. 4 may be due to lack of adjustment for resonance of the water column, or it may be due to the unstable water surface, in which event possibly the straight form of tube might give the better wave-form. Additional photographs are needed to test this point. The fundamental tone was about 132 vibrations per second.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
December, 1919.

OPTICAL PROBLEMS IN GLASSMAKING.[2]

BY H. P. GAGE.

IN its manufacture the physical properties of glass are of the highest importance, even more so than the chemical properties, for really good glass is a substance of great chemical inertness.

Optical problems which are met with in the manufacture of glass may be illustrated in the design of a form of glass which will produce a predetermined

[1] See abstract of paper on "Possible Standard of Sound,—Study of the Energy Of," Chicago Meeting of American Physical Society, November, 1919.

[2] Abstract of a paper presented at the St. Louis meeting of the American Physical Society, January 1, 1920.

optical effect such as is illustrated in the behavior of railway signal lenses and automobile headlamp glasses. The danger of unsuitable design of glass is illustrated in the danger of an obscuring reflection from colored railway signals produced by the powerful beams of electric locomotive headlamps. Such obscuring reflection is overcome by the use of curved instead of flat colored roundels in the railway signals.

A demonstration was made of colored glasses for railway signal and other purposes, among which may be mentioned pyromete rred glasses, pyrometer absorption glasses, eye protective goggles, glasses which are nearly clear but absorb ultra-violet, or absorb infra-red, and glasse opaques or nearly opaque which transmit ultra-violet, or transmit infra-red. A series of lantern slides was exhibited illustrating the transmission of many colored glasses as photographed by the Hilger wedge spectragraph. A demonstration was given in another room of the projection of spectra of railway signal and other colored glasses. Also demonstration was made of ultra-violet transmission and absorption of various glasses as shown by the fluorescence of an anthracene coated screen and the infra-red transmission of glasses as illustrated by the ignition of ordinary typewriter carbon paper.

THE FUNCTIONS OF INTENSITY AND PHASE IN THE BINAURAL LOCATION OF PURE TONES.[1]

BY G. W. STEWART.

THE permanence of the value of knowledge concerning the physical aspects of audition gives to contributions in this field a greater importance than is at first apparent. Experiments of the writer show (discussion limited to pure tones).

1. That intensity cannot be an important factor in binaural location at least for frequencies 256 to 1,024 d.v., the evidence being both qualitative and quantitative.

2. That the effect of phase, alone, is quantitatively as well as qualitatively in agreement with the statement that location is determined entirely by the phase effect with frequencies from 100 to 1,200 d.v.

3. The phase effect cannot be explained indirectly by the intensity effect.

The results furnish a basis for further work with complex tones.

PHYSICAL LABORATORY,
STATE UNIVEBSITY OF IOWA.

[1]Abstract of a paper presented at the St. Louis meeting of the American Physical Society, November 29, 1919.